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## Problem of the Month Problem 0: September 2022

- (a) Consider the integers 392, 487, 638, and 791. For each of these integers, do the following.
  - (i) Determine whether the integer is a multiple of 7.
  - (ii) With the hundreds digit equal to A, the tens digit equal to B, and the units digit equal to C, compute 2A + 3B + C.

What do you notice?

- (b) Suppose n = ABC is a three-digit integer (A is the hundreds digit, B is the tens digit, and C is the units digit). Show that if ABC is a multiple of 7, then 2A + 3B + C is a multiple of 7.
- (c) Show that if 2A + 3B + C is a multiple of 7, then the three-digit integer n = ABC is a multiple of 7.
- (d) Suppose ABCDEF is a six-digit integer that has each of its digits different from 0. Show that ABCDEF is a multiple of 7 if and only if BCDEFA is a multiple of 7.
- (e) Think of ways to generalize the fact in part (d).