Problem of the Month

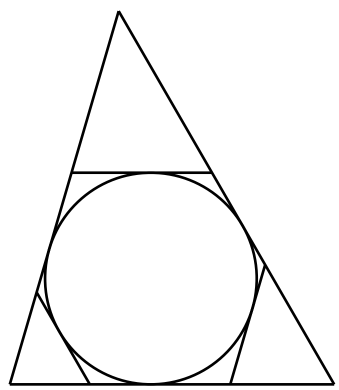
Problem 1: October 2022

In any triangle, there is a unique circle called its *incircle* that can be drawn in such a way that it is tangent to all three sides of the triangle. For a given triangle, the radius of its incircle is known as its *inradius* and is denoted by r.

For each side of the triangle (which is tangent to the incircle), another tangent to the incircle can be drawn in such a way that it is parallel to that side. The three sides as well as these three new tangents give a total of six tangents to the incircle. They uniquely determine a hexagon that we will call the *Seraj hexagon* of the triangle.

Finally, for a given triangle, we will denote by s its *semiperimeter*, which is defined to be half of its perimeter.

The diagram below is of a triangle showing its incircle and Seraj hexagon.



(a) Sketch the 3–4–5 triangle with its incircle and Seraj hexagon. Compute its inradius, semiperimeter, and the area of its Seraj hexagon.

(b) Find a general expression for the area of a triangle in terms only of its inradius and semiperimeter.

(c) Find a general expression for the area of the Seraj hexagon of a triangle in terms of its three side lengths, its semiperimeter, and its inradius.

(d) What is the largest possible value that can be obtained by dividing the area of a triangle's Seraj hexagon by the total area of the triangle?