Problem of the Month
Problem 1: October 2022

Hint

(a) There are several ways to compute the area of the Seraj hexagon. One is to subtract the areas of three smaller triangles from that of the full triangle.

(b) From the centre of the incircle, draw a radius to each point of tangency the circle has with the triangle.

(c)
- If $\triangle ABC$ is similar to $\triangle DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. It is a useful general fact that if we denote this common ratio by k, then any two corresponding altitudes of these two triangles also have a ratio of k. Can you compute the ratio of the areas of $\triangle ABC$ and $\triangle DEF$ in terms of k?
- One possible expression is

$$rs\left[1 - \left(1 - \frac{a}{s}\right)^2 - \left(1 - \frac{b}{s}\right)^2 - \left(1 - \frac{c}{s}\right)^2\right]$$

(d) Try to prove that $3(x^2 + y^2 + z^2) \geq (x + y + z)^2$ is true for all real numbers x, y, and z and determine a condition on x, y, and z that implies $3(x^2 + y^2 + z^2) = (x + y + z)^2$.