# The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

***cemc.uwaterloo.ca***

# *2022 Gauss Contests*

**(Grades 7 and 8)**

**Wednesday, May 18, 2022**
**(in North America and South America)**

**Thursday, May 19, 2022**
**(outside of North America and South America)**

*Solutions*

***Centre for Education in Mathematics and Computing Faculty and Staff***

Ed Anderson
Jeff Anderson
Terry Bae
Jacquelene Bailey
Shane Bauman
Ersal Cahit
Diana Castañeda Santos
Sarah Chan
Ashely Congi
Serge D'Alessio
Fiona Dunbar
Mike Eden
Sandy Emms
Barry Ferguson
Steve Furino
Lucie Galinon
Robert Garbary
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Sandy Graham
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Comfort Mintah
Jen Nelson
Ian Payne
J.P. Pretti
Alexandra Rideout
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Troy Vasiga
Christine Vender
Heather Vo
Bonnie Yi

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Rachael Verbruggen, University of Waterloo, Waterloo, ON
Robert Wong, Edmonton, AB
Chris Wu, Ledbury Park E. and M.S., Toronto, ON
Lori Yee, William Dunbar P.S., Pickering, ON

**Grade 7**

1. Arranging the five answers along with 10 from smallest to largest, we get $1, 5, 8, 10, 13, 19$. Since 10 is 2 more than 8 and 10 is 3 less than 13, then 8 is the closest number to 10.

   Answer: (C)

2. Reading from the graph, the greatest number of hours that Gabe spent riding his bike is 4, and this occurred on Tuesday.

   Answer: (B)

3. Of the given answers, 0 is the only value of $x$ that is less than 5.

   Answer: (B)

4. Since $18 + 5 = 23$, and $23 + 5 = 28$, and $28 + 5 = 33$, the next three terms in the sequence are $23, 28, 33$.

   Answer: (C)

5. The faces shown are labelled with $1, 3$ and 5 dots. Therefore, the other three faces are labelled with $2, 4$ and 6 dots. The total number of dots on the other three faces is $2 + 4 + 6 = 12$.

   Answer: (D)

6. Since $\angle ABC$ measures $90^\circ$, and $\angle ABC = 44^\circ + x^\circ$, then $x = 90 - 44 = 46$.

   Answer: (A)

7. The largest height of the singers in Saura's choir is 183.5 cm.
   The smallest height of the singers in Saura's choir is 141 cm.
   Thus, the range of their heights is $183.5$ cm $-$ $141$ cm $= 42.5$ cm.

   Answer: (A)

8. Beginning at the origin $(0, 0)$, the point $(3, -4)$ is located right 3 units and down 4 units.
   In the diagram, the point $(3, -4)$ is labelled $T$.

   Answer: (E)

9. When Emily jumps for 75 seconds, she jumps for $60 + 15$ seconds.
   Jumping at the rate of 52 times in 60 seconds, Emily jumps $52 \div 4 = 13$ times in $60 \div 4 = 15$ seconds.
   Since Emily jumps 52 times in 60 seconds and 13 times in 15 seconds, then Emily jumps $52 + 13 = 65$ times in $60 + 15 = 75$ seconds.

   Answer: (C)

10. In \$1.00 worth of dimes, there are $\dfrac{\$1.00}{\$0.10} = 10$ dimes.

    In \$1.00 worth of quarters, there are $\dfrac{\$1.00}{\$0.25} = 4$ quarters.

    The jar contains 10 dimes and a total of $10 + 4 = 14$ coins.

    If Terry randomly removes one coin from the jar, the probability that it is a dime is $\dfrac{10}{14} = \dfrac{5}{7}$.

    Answer: (E)

11. Since 42 is an even number, then 2 is a factor of 42.
Since $42 = 2 \times 21$ and $21 = 3 \times 7$, then $42 = 2 \times 3 \times 7$.
Each of $2, 3$ and 7 is a prime number and each is a factor of 42.
Thus, the sum of the prime factors of 42 is $2 + 3 + 7 = 12$.
(We note that $1, 6, 14, 21$, and 42 are also factors of 42, however they are not prime numbers.)
ANSWER: (C)

12. $\triangle PQR$ is isosceles with $PQ = PR$, and so $\angle PRQ = \angle PQR$.
The sum of the angles in $\triangle PQR$ is $180^\circ$.
Since $\angle QPR = 70^\circ$, then the other two angles in this triangle add to $180^\circ - 70^\circ = 110^\circ$.
Since these two angles are equal, they each measure $110^\circ \div 2 = 55^\circ$, and so $x = 55$.
Since $QRST$ is a rectangle, each of its interior angles is a right angle, and so $y = 90$.
The value of $x + y$ is $55 + 90 = 145$.
ANSWER: (D)

13. A two-digit number has at least one digit that is a 4 if its tens digit is a 4 or if its ones digit is a 4.
There are 10 two-digit numbers whose tens digit is a 4.
These are $40, 41, 42, 43, 44, 45, 46, 47, 48$, and 49.
There are 9 two-digit numbers whose ones digit is a 4.
These are $14, 24, 34, 44, 54, 64, 74, 84, 94$, including 44 which was counted in our previous list.
The number of two-digit numbers that have at least one digit that is a 4 is $10 + 9 - 1 = 18$.
ANSWER: (C)

14. The side lengths of each of the three identical squares are equal.
The perimeter of $WXYZ$ is 56 m and is comprised of 8 such side lengths.
Thus, the length of each side of the three identical squares is $56 \text{ m} \div 8 = 7$ m.
The area of each of the three identical squares is $7 \text{ m} \times 7 \text{ m} = 49 \text{ m}^2$.
Therefore, the area of $WXYZ$ is $3 \times 49 \text{ m}^2 = 147 \text{ m}^2$.
ANSWER: (B)

15. The first Wednesday of a month must occur on one of the first 7 days of a month, that is, the 1$^{\text{st}}$, 2$^{\text{nd}}$, 3$^{\text{rd}}$, 4$^{\text{th}}$, 5$^{\text{th}}$, 6$^{\text{th}}$, or 7$^{\text{th}}$ of the month.
The second Wednesday of a month occurs 7 days following the first Wednesday of that month, and the third Wednesday of a month occurs 14 days following the first Wednesday of that month.
Adding 14 days to each of the possible dates for the first Wednesday of the month, we get that the third Wednesday of a month must occur on the 15$^{\text{th}}$, 16$^{\text{th}}$, 17$^{\text{th}}$, 18$^{\text{th}}$, 19$^{\text{th}}$, 20$^{\text{th}}$, or 21$^{\text{st}}$ of that month.
Of the given answers, the public holiday cannot occur on the 22$^{\text{nd}}$ of that month.
ANSWER: (B)

16. *Solution 1*
When a standard fair coin is tossed three times, there are 8 possible outcomes.
If H represents head and T represents tail, these 8 outcomes are: HHH, HHT, HTH, THH, HTT, THT, TTH, and TTT.
Of these 8 outcomes, there are exactly 2 whose outcomes are all the same (HHH and TTT).
Therefore, the probability that the three outcomes are all the same is $\frac{2}{8} = \frac{1}{4}$.

*Solution 2*

The three outcomes are all the same exactly when each of the three tosses is a head or when each of the three tosses is a tail.

When a coin is tossed, the probability that the coin shows a head is $\frac{1}{2}$ and the probability that the coin shows a tail is also $\frac{1}{2}$ (there are two possible outcomes and each is equally probable).

Thus, the probability that each of the three tosses is a head is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Similarly, the probability that each of the three tosses is a tail is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Therefore, the probability that the three outcomes are all the same is $\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$.

Answer: (C)

17. If the value of $P$ is less than 9, then $QR + PPP + PPP$ is at most $99 + 888 + 888 = 1875$.
Since the given sum is 2022, then the value of $P$ cannot be less than 9 and thus must equal 9.
When $P = 9$, we get $QR + 999 + 999 = 2022$, and so $QR = 2022 - 999 - 999 = 24$.
Therefore $Q = 2$, $R = 4$, and so $P + Q + R = 9 + 2 + 4 = 15$.

Answer: (C)

18. We begin by recognizing that moving a block from Box A to Box B, and then moving the same block from Box B back to Box A has does not change the total mass in each box.
Since such a pair of moves increases the number of blocks that Jasmine moves from Box A to Box B, then all such pairs of moves will not occur in counting the fewest number of blocks that Jasmine could have moved from Box A to Box B.
(Similarly, moving a block from Box B to Box A, and then moving that same block from Box A back to Box B is a pair of moves that will not occur.)
That is, once a block has been moved from one box to another, that block will not be moved in the opposite direction between the two boxes.

Next, we consider what total masses of blocks can be moved from Box B to Box A.
Box B contains one 50 g block and three 10 g blocks, and so Jasmine can move the following total masses of blocks from Box B to Box A: 10 g, 20 g, 30 g, 50 g, 60 g, 70 g, and 80 g. (Can you see how to get each of these masses using the blocks in Box B and why no other masses are possible?)
If Jasmine moves 10 g from Box B to Box A, then she must move blocks whose total mass is 65 g + 10 g = 75 g from Box A to Box B so that Box A contains 65 g less than it did originally and Box B contains 65 g more than it did originally.
For each of the other masses that may be moved from Box B to Box A, we summarize the total mass of blocks that Jasmine must move from Box A to Box B. Note that in each case, the mass moved from Box A to Box B must be 65 g greater than the mass moved from Box B to Box A.

| Mass moved from Box B to Box A | Mass moved from Box A to Box B |
|---|---|
| 10 g | 75 g |
| 20 g | 85 g |
| 30 g | 95 g |
| 50 g | 115 g |
| 60 g | 125 g |
| 70 g | 135 g |
| 80 g | 145 g |

Next, we determine if it is possible for Jasmine to choose blocks from Box A whose total mass is given in the second column in the table above.
Recall that Box A originally contains one 100 g block, one 20 g block, and three 5 g blocks.
Is it possible for Jasmine to choose blocks from Box A whose total mass is exactly 75 g?
Since the 100 g block is too heavy, and the remaining blocks have a total mass that is less than 75 g, then it is not possible for Jasmine to choose blocks from Box A whose total mass is exactly 75 g.
In the table below, we determine which of the exact total masses Jasmine is able to move from Box A to Box B.
For those masses which are possible, we list the number of blocks that Jasmine must move from Box A to Box B.

| Total mass moved from Box B to Box A | Total mass moved from Box A to Box B | Is it possible to move this mass from Box A to Box B? | Number of blocks moved from Box A to Box B |
|---|---|---|---|
| 10 g | 75 g | No | |
| 20 g | 85 g | No | |
| 30 g | 95 g | No | |
| 50 g | 115 g | Yes; one 100 g, three 5 g | 4 |
| 60 g | 125 g | Yes; one 100 g, one 20 g, one 5 g | 3 |
| 70 g | 135 g | Yes; one 100 g, one 20 g, three 5 g | 5 |
| 80 g | 145 g | No | |

From the table above, the fewest number of blocks that Jasmine could have moved from Box A to Box B is 3. In this case, Jasmine moves a total mass of $1(100\text{ g}) + 1(20\text{ g}) + 1(5\text{ g}) = 125$ g from Box A to Box B and she moves a total mass of $1(50\text{ g}) + 1(10\text{ g}) = 60$ g from Box B to Box A.
This confirms that Box B contains 125 g − 60 g = 65 g more than it did originally (and hence Box A contains 65 g less), as required.

Answer: (A)

19. The original ratio of red candies to blue candies is $3 : 5$, and so the number of red candies was a positive integer multiple of 3, and the number of blue candies was the same positive integer multiple of 5.
For example, there could have been $3 \times 1 = 3$ red candies and $5 \times 1 = 5$ blue candies, or $3 \times 2 = 6$ red candies and $5 \times 2 = 10$ blue candies, or $3 \times 3 = 9$ red candies and $5 \times 3 = 15$ blue candies, and so on.
We continue these possibilities in the table below, and consider the number of red and blue candies and the resulting ratio after three blue candies are removed from the dish.

| Possible numbers of red and blue candies originally | 3 red, 5 blue | 6 red, 10 blue | 9 red, 15 blue | 12 red, 20 blue | 15 red, 25 blue | 18 red, 30 blue |
|---|---|---|---|---|---|---|
| Numbers of red and blue candies after 3 blue are removed | 3 red, 2 blue | 6 red, 7 blue | 9 red, 12 blue | 12 red, 17 blue | 15 red, 22 blue | 18 red, 27 blue |
| New ratio of the number of red candies to blue | $3 : 2$ | $6 : 7$ | $9 : 12$ $= 3 : 4$ | $12 : 17$ | $15 : 22$ | $18 : 27$ $= 2 : 3$ |

If the dish originally contained 18 red candies and 30 blue (note that $18 : 30 = 3 : 5$), then when three blue candies are removed, the ratio of the number of red candies to blue candies

becomes $18 : 27$ which is equal to $2 : 3$, as required.
Therefore, there were $30 - 18 = 12$ more blue candies than red candies in the dish before any candies were removed.

ANSWER: (B)

20. Let Anyu, Brad, Chi, and Diego be represented by $A$, $B$, $C$, and $D$, respectively, and so their original order is $ABCD$.
When rearranged, $A$ is not in the $1^{\text{st}}$ position, and so there are exactly 3 cases to consider: $A$ is in the $2^{\text{nd}}$ position, or $A$ is in the $3^{\text{rd}}$ position, or $A$ is in the $4^{\text{th}}$ position.
For each of these 3 cases, we count the number of ways to arrange $B$, $C$, and $D$.

Case 1: $A$ is in the $2^{\text{nd}}$ position
Since $A$ is in the $2^{\text{nd}}$ position, $B$ can be in any of the other 3 positions ($1^{\text{st}}, 3^{\text{rd}}$ or $4^{\text{th}}$).
If $B$ is in the $1^{\text{st}}$ position, then there is exactly one possible rearrangement: $BADC$ (since $C$ and $D$ cannot be in the $3^{\text{rd}}$ and $4^{\text{th}}$ positions respectively).
If $B$ is in the $3^{\text{rd}}$ position, then there is exactly one possible rearrangement: $DABC$ (since $D$ cannot be in the $4^{\text{th}}$ position).
If $B$ is in the $4^{\text{th}}$ position, then there is exactly one possible rearrangement: $CADB$ (since $C$ cannot be in the $3^{\text{rd}}$ position).
Thus there are exactly 3 possible rearrangements when $A$ is in the $2^{\text{nd}}$ position.

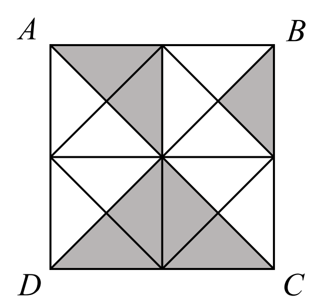
Case 2: $A$ is in the $3^{\text{rd}}$ position
Since $A$ is in the $3^{\text{rd}}$ position, $C$ can be in any of the other 3 positions.
In a manner similar to Case 1, it can be shown that there are 3 possible rearrangements in this case: $CDAB$, $DCAB$, and $BDAC$.

Case 3: $A$ is in the $4^{\text{th}}$ position
Since $A$ is in the $4^{\text{th}}$ position, $D$ can be in any of the other 3 positions.
Similarly, there are 3 possible rearrangements in this case: $DCBA$, $CDBA$, and $BCDA$.

So that each person is not in their original position, the four friends can rearrange themselves in $3 + 3 + 3 = 9$ different ways.
(Such a rearrangement of a list in which no element appears in its original position is called a *derangement*.)

ANSWER: (B)

21. We begin by constructing the three diagonals inside the smaller squares that were missing from the diagram given in the question, as shown.
The two diagonals inside each of these 4 smaller squares divide each smaller square into 4 identical triangles having equal area.
Thus, square $ABCD$ is divided into $4 \times 4 = 16$ such triangles.
Since 7 of these triangles are shaded, then the fraction of $ABCD$ that is shaded is $\frac{7}{16}$.




ANSWER: (C)

22. *Solution 1*
Since the sum of $p, q, r, s$ and the sum of $q, r, s, t$ are both equal to 35, and $q, r, s$ are added in each sum, then $p = t$.
Similarly, since the sum of $q, r, s, t$ and the sum of $r, s, t, u$ are both equal to 35, and $r, s, t$ are added in each sum, then $q = u$.

It can similarly be shown that $r = v$ and $s = w$.
Using the above observations, the sequence $p, q, r, s, t, u, v, w$ can be written as $p, q, r, s, p, q, r, s$.
Since the sum of $q$ and $v$ is 14 and $r = v$, then the sum of $q$ and $r$ is 14.
Since the sum of $p, q, r, s$ is 35 and the sum of $q$ and $r$ is 14, then the sum of $p$ and $s$ is $35 - 14 = 21$.
The value of $p$ is as large as possible exactly when the value of $s$ is as small as possible.
Since $s$ is a positive integer, its smallest possible value is 1.
Therefore, the largest possible value of $p$ is $21 - 1 = 20$.
(We note that $20, 4, 10, 1, 20, 4, 10, 1$ is an example of such a list.)

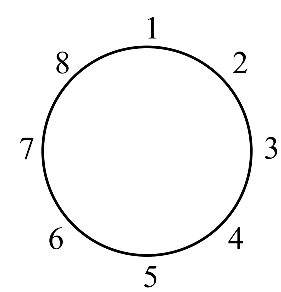
*Solution 2*
The sum of the values of each group of four consecutive letters is 35.
Thus, $p+q+r+s = 35$ and $t+u+v+w = 35$, and so $(p+q+r+s)+(t+u+v+w) = 35+35 = 70$.
Rearranging the sum of these eight letters, we get

$$p + q + r + s + t + u + v + w = (p + w) + (q + v) + (r + s + t + u) = 70$$

However, $r + s + t + u = 35$ (the sum of the values of four consecutive letters), and $q + v = 14$.
Substituting, we get $(p + w) + 14 + 35 = 70$, and so $p + w = 21$.
The value of $p$ is as large as possible exactly when the value of $w$ is as small as possible.
Since $w$ is a positive integer, its smallest possible value is 1.
Substituting, we get $p + 1 = 21$, and so the largest possible value of $p$ is 20.
(We note that $20, 12, 2, 1, 20, 12, 2, 1$ is an example of such a list.)

Answer: (C)

23. Katharina placed the 8 letters in a mixed-up order around the circle.
We number the position of $L$ at the top the circle as 1, the next number moving clockwise from $L$ position 2, and so on through to position 8, as shown.
Jaxon begins at position 1, writes down the letter $L$, and moves clockwise writing down every third letter that he has not yet written down.
Thus, the first 3 letters that Jaxon writes down are the letters in positions 1, 4 and 7.




Continuing to move clockwise around the circle, the next three positions at which letters have not yet been written down are positions 8, 2 and 3 (the letter in position 1 was already written down), and so Jaxon writes down the letter in position 3.
The next three positions at which letters have not yet been written down are 5, 6 and 8 (the letters in positions 4 and 7 were already written down), and so Jaxon writes down the letter in position 8.
At this point, Jaxon has written the letters in positions 1, 4, 7, 3, and 8.
The next three positions at which letters have not yet been written down are 2, 5 and 6 (the letters in positions 1, 3 and 4 were already written down), and so Jaxon writes down the letter in position 6.
At this point, only the letters in positions 2 and 5 have not been written.
Since Jaxon left off at position 6, he skips the letter at position 2, skips the letter at position 5, and then writes down the letter at position 2.
Finally, Jaxon writes the final letter at position 5.
Thus in order, Jaxon writes the letters in positions 1, 4, 7, 3, 8, 6, 2, and 5.

Since Jaxon's list is $L, M, N, O, P, Q, R, S$, then $L$ is the letter in position 1 of Katharina's ordering, $M$ is the letter in position 4, $N$ is the letter in position 7, and so on.
Therefore, Katharina's clockwise order is $L, R, O, M, S, Q, N, P$.

ANSWER: (C)

24. A palindrome greater than 10 000 and less than 100 000 is a 5-digit positive integer of the form $abcba$, where $a, b$ and $c$ are digits and $a \neq 0$.
A positive integer is a multiple of 18 if it is a multiple of both 2 and 9 (and a positive integer that is a multiple of both 2 and 9 is a multiple of 18).
A positive integer is a multiple of 2 if it is even, and thus the digit $a$ is equal to $2, 4, 6$ or $8$ (recall $a \neq 0$).
A positive integer is a multiple of 9 exactly when the sum of its digits is a multiple of 9, and thus $a + b + c + b + a$ or $2a + 2b + c$ is a multiple of 9.
Next we consider four possible cases, one case for each of the possible values of $a$.

<u>Case 1</u>: $a = 2$
When $a = 2$, we require that $2a + 2b + c = 4 + 2b + c$ be a multiple of 9.
Since $4 + 2b + c \geq 4$, then the smallest possible multiple of 9 that $4 + 2b + c$ can equal is 9.
Since $b \leq 9$ and $c \leq 9$, then $4 + 2b + c$ is at most $4 + 2(9) + 9 = 31$.
Thus, $4 + 2b + c$ can equal $9, 18$ or $27$, which gives $2b + c$ equal to $5, 14$ or $23$ respectively.
Next, we determine the possible values of $b$ and $c$ so that $2b + c$ is equal to $5, 14$ or $23$.

| $2b + c = 5$ | $2b + c = 14$ | $2b + c = 23$ |
|---|---|---|
| $b = 2, c = 1$ | $b = 7, c = 0$ | $b = 9, c = 5$ |
| $b = 1, c = 3$ | $b = 6, c = 2$ | $b = 8, c = 7$ |
| $b = 0, c = 5$ | $b = 5, c = 4$ | $b = 7, c = 9$ |
| | $b = 4, c = 6$ | |
| | $b = 3, c = 8$ | |

Thus when $a = 2$, there are $3 + 5 + 3 = 11$ such palindromes.

<u>Case 2</u>: $a = 4$
When $a = 4$, we require that $2a + 2b + c = 8 + 2b + c$ be a multiple of 9.
Since $8 + 2b + c \geq 8$, then the smallest possible multiple of 9 that $8 + 2b + c$ can equal is 9.
Since $b \leq 9$ and $c \leq 9$, then $8 + 2b + c$ is at most $8 + 2(9) + 9 = 35$.
Thus, $8 + 2b + c$ can equal $9, 18$ or $27$, which gives $2b + c$ equal to $1, 10$ or $19$ respectively.
Next, we determine the possible values of $b$ and $c$ so that $2b + c$ is equal to $1, 10$ or $19$.

| $2b + c = 1$ | $2b + c = 10$ | $2b + c = 19$ |
|---|---|---|
| $b = 0, c = 1$ | $b = 5, c = 0$ | $b = 9, c = 1$ |
| | $b = 4, c = 2$ | $b = 8, c = 3$ |
| | $b = 3, c = 4$ | $b = 7, c = 5$ |
| | $b = 2, c = 6$ | $b = 6, c = 7$ |
| | $b = 1, c = 8$ | $b = 5, c = 9$ |

Thus when $a = 4$, there are $1 + 5 + 5 = 11$ such palindromes.

<u>Case 3</u>: $a = 6$
When $a = 6$, we require that $2a + 2b + c = 12 + 2b + c$ be a multiple of 9.
Since $12 + 2b + c \geq 12$ and $12 + 2b + c \leq 12 + 2(9) + 9 = 39$, then $12 + 2b + c$ can equal 18, 27 or 36, which gives $2b + c$ equal to 6, 15 or 24, respectively.

| $2b+c=6$ | $2b+c=15$ | $2b+c=24$ |
|---|---|---|
| $b=3,c=0$ | $b=7,c=1$ | $b=9,c=6$ |
| $b=2,c=2$ | $b=6,c=3$ | $b=8,c=8$ |
| $b=1,c=4$ | $b=5,c=5$ | |
| $b=0,c=6$ | $b=4,c=7$ | |
| | $b=3,c=9$ | |

Thus when $a=6$, there are $4+5+2=11$ such palindromes.

Case 4: $a=8$
When $a=8$, we require that $2a+2b+c=16+2b+c$ be a multiple of 9.
Since $16+2b+c\geq 16$ and $16+2b+c\leq 16+2(9)+9=43$, then $16+2b+c$ can equal 18, 27 or 36, which gives $2b+c$ equal to 2, 11 or 20, respectively.

| $2b+c=2$ | $2b+c=11$ | $2b+c=20$ |
|---|---|---|
| $b=1,c=0$ | $b=5,c=1$ | $b=9,c=2$ |
| $b=0,c=2$ | $b=4,c=3$ | $b=8,c=4$ |
| | $b=3,c=5$ | $b=7,c=6$ |
| | $b=2,c=7$ | $b=6,c=8$ |
| | $b=1,c=9$ | |

Thus when $a=8$, there are $2+5+4=11$ such palindromes.
Therefore, the number of palindromes that are greater than 10 000 and less than 100 000 and that are multiples of 18 is $11+11+11+11=44$.

ANSWER: (D)

25. After all exchanges there are 4 balls in each bag, and so if a bag contains exactly 3 different colours of balls, then it must contain exactly 2 balls of the same colour and 2 balls each having a colour that is different than all other balls in the bag.
Among the 8 balls in the two bags, there are 2 red balls and 2 black balls and each of the remaining balls has a colour that is different than all the other balls.
Thus after all exchanges, one bag must contain the 2 red balls and the other bag must contain the 2 black balls.
Since Becca's bag initially contains both black balls, and Becca moves only 1 ball from her bag to Arjun's bag, it is not possible for Arjun's bag to contain the 2 black balls after all exchanges.
This tells us that if each bag contains exactly 3 different colours of balls after all exchanges, then Arjun's bag contains the 2 red balls and Becca's bag contains the 2 black balls.
We let the first letter of each colour represent a ball of that colour.
Then initially, Arjun's bag contains $RRGYV$ and Becca's bag contains $BBO$.
For the first ball chosen to be moved, there are exactly two cases to consider:
Case 1: The first ball moved from Arjun's bag to Becca's bag is $R$, or
Case 2: The first ball moved from Arjun's bag to Becca's bag is not $R$ (thus it is $G$, $Y$ or $V$).
We begin with Case 1 and determine the probability that after all exchanges, each bag contains exactly 3 different colours of balls.

Case 1: The first ball moved from Arjun's bag to Becca's bag is $R$

Since Arjun's bag initially contains 5 balls, 2 of which are $R$, the probability that $R$ is chosen as the first ball to move is $\frac{2}{5}$.
After $R$ is moved from Arjun's bag to Becca's, Arjun's bag contains $RGYV$ and Becca's bag

contains $BBOR$.
Since Arjun's bag must contain both $R$'s after all exchanges, and Becca moves only 1 ball from her bag to Arjun's, then the next ball chosen to move must be $R$.
Becca's bag contains 4 balls, 1 of which is $R$, and so in this case the probability that $R$ is chosen as the second ball to move is $\frac{1}{4}$.
After the first two balls are moved, Arjun's bag contains $RRGYV$ and Becca's bag contains $BBO$.
Finally, a ball is chosen from Arjun's bag and moved to Becca's.
Since Arjun's bag must contain both $R$'s after all exchanges, then the ball that is chosen must be $G$, $Y$ or $V$ which occurs with probability $\frac{3}{5}$.
If the first ball moved from Arjun's bag to Becca's bag is $R$, then the probability that each bag contains exactly 3 different colours of balls after all exchanges is given by the product of the individual probabilities of choosing each of the 3 balls or $\frac{2}{5} \times \frac{1}{4} \times \frac{3}{5} = \frac{6}{100} = \frac{3}{50}$.

Case 2: The first ball moved from Arjun's bag to Becca's bag is $G$, $Y$ or $V$

Since Arjun's bag initially contains 5 balls, the probability that $G$, $Y$ or $V$ is chosen as the first ball to move is $\frac{3}{5}$.
After one of $G$, $Y$ or $V$ is moved from Arjun's bag to Becca's, Arjun's bag contains $RR$ and two of $G$, $Y$ and $V$, and Becca's bag contains $BBO$ and one of $G$, $Y$ or $V$.
For the second ball chosen to be moved, there are exactly two cases to consider and so we split Case 2 into these two separate cases.
Case 2a: The first ball moved from Arjun's bag to Becca's bag is $G$, $Y$ or $V$, and the ball moved from Becca's bag to Arjun's bag is $B$, or
Case 2b: The first ball moved from Arjun's bag to Becca's bag is $G$, $Y$ or $V$, and the ball moved from Becca's bag to Arjun's bag is not $B$ (that is, it is $O$ or the first ball moved).
We begin with Case 2a and determine the probability that after all exchanges, each bag contains exactly 3 different colours of balls.

Case 2a: The first ball moved from Arjun's bag to Becca's bag is $G$, $Y$ or $V$, and the ball moved from Becca's bag to Arjun's bag is $B$
We previously determined that the probability that $G$, $Y$ or $V$ is chosen as the first ball to move is $\frac{3}{5}$.
At this point, Becca's bag contains 4 balls, 2 of which are $B$'s, and so the probability that $B$ is chosen is $\frac{2}{4} = \frac{1}{2}$.
After $B$ is moved from Becca's bag to Arjun's, Arjun's bag contains $RRB$ and two of $G$, $Y$ and $V$ and Becca's bag contains $BO$ and one of $G$, $Y$ or $V$.
Since Becca's bag must contain both $B$'s after all exchanges, then the final ball chosen to move must be $B$.
Arjun's bag contains 5 balls, 1 of which is $B$, and so in this case the probability that $B$ is chosen as the final ball to move is $\frac{1}{5}$.
If the first ball moved from Arjun's bag to Becca's bag is $G$, $Y$ or $V$, and the ball moved from Becca's bag to Arjun's bag is $B$, then the probability that each bag contains exactly 3 different colours of balls after all exchanges is given by the product of the individual probabilities of choosing each of the 3 balls or $\frac{3}{5} \times \frac{1}{2} \times \frac{1}{5} = \frac{3}{50}$.

Case 2b: The first ball moved from Arjun's bag to Becca's bag is $G$, $Y$ or $V$, and the ball moved from Becca's bag to Arjun's bag is $O$ or the first ball moved ($G$, $Y$ or $V$)
The probability that $G$, $Y$ or $V$ is chosen as the first ball to move is $\frac{3}{5}$.
At this point, Becca's bag contains 4 balls, 2 of which are $B$, and so the probability that $B$ is

not chosen is $\frac{2}{4} = \frac{1}{2}$.
After this ball ($O$, $G$, $Y$ or $V$) is moved from Becca's bag to Arjun's, Arjun's bag contains $RR$ and three of $O$, $G$, $Y$ and $V$ and Becca's bag contains two $B$'s and one of $O$, $G$, $Y$ or $V$.
Since Arjun's bag must contain both $R$'s after all exchanges, then the final ball chosen to move must be one of the 3 balls in Arjun's bag that is not $R$.
Arjun's bag contains 5 balls, 2 of which are $R$, and so in this case the probability that $R$ is not chosen as the final ball to move is $\frac{3}{5}$.
If the first ball moved from Arjun's bag to Becca's bag is $G$, $Y$ or $V$, and the ball moved from Becca's bag to Arjun's bag is not $B$, then the probability that each bag contains exactly 3 different colours of balls after all exchanges is given by the product of the individual probabilities of choosing each of the 3 balls or $\frac{3}{5} \times \frac{1}{2} \times \frac{3}{5} = \frac{9}{50}$.

Finally, the probability that each bag contains exactly 3 different colours of balls after all exchanges is $\frac{3}{50} + \frac{3}{50} + \frac{9}{50} = \frac{15}{50} = \frac{3}{10}$.

Answer: (A)

**Grade 8**

1. The regular pentagon shown has 5 sides, each with length 2 cm.
The perimeter of the pentagon is $5 \times 2$ cm $= 10$ cm.

ANSWER: (E)

2. The faces shown are labelled with $1, 3$ and 5 dots. Therefore, the other three faces are labelled with $2, 4$ and 6 dots. The total number of dots on the other three faces is $2 + 4 + 6 = 12$.

ANSWER: (D)

3. If the number is $n$, then "a number increased by five" is best represented by the expression $n + 5$. The equation that best represents "a number increased by five equals 15" is $n + 5 = 15$.

ANSWER: (C)

4. Reading from the graph, the approximate number of bobbleheads sold in the years 2016, 2017, 2018, 2019, 2020, and 2021 were 20, 35, 40, 38, 60, and 75, respectively.
Beginning with 2016 and 2017, the increases (or decreases) in the sale of bobbleheads between consecutive years are approximately $35 - 20 = 15$, $40 - 35 = 5$, $38 - 40 = -2$ (a decrease of 2), $60 - 38 = 22$, and $75 - 60 = 15$.
Thus the greatest increase in the sale of bobbleheads was approximately 22 and this occurred between 2019 and 2020.

ANSWER: (D)

5. Continuing to count down by 11, we get

$$72, 61, 50, 39, 28, 17, 6, -5, \ldots$$

The last number that Aryana will count that is greater than 0 is 6.

ANSWER: (C)

6. Since $\angle ABC = 90^\circ$, then $44 + x + x = 90$ or $2x = 90 - 44$, and so $2x = 46$ or $x = 23$.

ANSWER: (B)

7. Since $\frac{5}{4} = 1\frac{1}{4}$ and $1^2 = 1$, then each of the choices (A) $-1$, (B) $\frac{5}{4}$ and (C) $1^2$ is at least 1 away from 0.
Since $-\frac{4}{5} = -0.8$ and $-0.8$ is closer to 0 than 0.9 is to 0, then $-\frac{4}{5}$ is the closest value to zero.

ANSWER: (D)

8. *Solution 1*
The value of 4 quarters is \$1.00.
Thus, the value of $4 \times 100 = 400$ quarters is $\$1.00 \times 100 = \$100.00$.
Since the jar initially contains 267 quarters, then $400 - 267 = 133$ quarters must be added to the jar for the total value of the quarters to equal \$100.00.

*Solution 2*
The value of 267 quarters is $\$0.25 \times 267 = \$66.75$ and so $\$100.00 - \$66.75 = \$33.25$ must be added to the jar so that it contains \$100.00.
Thus, the number of quarters that must be added to the jar is $\dfrac{\$33.25}{\$0.25} = 133$.

ANSWER: (D)

9. Each package of greeting cards comes with $10 - 8 = 2$ more envelopes than cards.
Thus, 3 packages of greeting cards comes with $3 \times 2 = 6$ more envelopes than cards, and 4 packages of greeting cards comes with $4 \times 2 = 8$ more envelopes than cards.
Kirra began with 7 cards and no envelopes.
To have more envelopes than cards, Kirra must buy enough packages to make up the difference between the number of cards and the number of envelopes (which is 7).
Thus, the smallest number of packages that Kirra must buy is 4.
Note: We can check that if Kirra buys 3 packages she has $3 \times 8 + 7 = 31$ cards and $3 \times 10 = 30$ envelopes, and thus fewer envelopes than cards. However, if she buys 4 packages, she has $4 \times 8 + 7 = 39$ cards and $4 \times 10 = 40$ envelopes, and thus more envelopes than cards, as required.

ANSWER: (B)

10. The horizontal distance between the point $(a, b)$ and the $y$-axis is $a$ units, and the horizontal distance between the point $(c, d)$ and the $y$-axis is $c$ units.
Since $(a, b)$ is horizontally farther from the $y$-axis than $(c, d)$ is from the $y$-axis, then $a > c$ and so statement (E) is true.

Let us consider why each of the other statements is false.
The points $(a, b)$ and $(c, d)$ each lie above the $x$-axis and thus $b > 0$ and $d > 0$.
The point $(e, f)$ lies below the $x$-axis and thus $f < 0$.
Since $b > 0$ and $f < 0$, then $b > f$ and so statement (C) is false.
Further, the vertical distance between the point $(a, b)$ and the $x$-axis is $b$ units, and the vertical distance between the point $(c, d)$ and the $x$-axis is $d$ units.
Since $(a, b)$ is vertically farther from the $x$-axis than $(c, d)$ is from the $x$-axis, then $b > d$ and so statement (B) is false.

Similarly, the points $(a, b)$ and $(c, d)$ each lie to the right of the $y$-axis and thus $a > 0$ and $c > 0$.
The point $(e, f)$ lies to the left of the $y$-axis and thus $e < 0$.
Since $a > 0$ and $e < 0$, then $a > e$ and so statement (D) is false.
Since $c > 0$ and $e < 0$, then $c > e$ and so statement (A) is also false.

ANSWER: (E)

11. In the sequence, the letters of the alphabet repeat in blocks of 26 letters.
Thus, 10 of these blocks gives a sequence that contains $10 \times 26 = 260$ letters.
Each block of 26 letters ends with the letter $Z$, and so the $260^{\text{th}}$ letter is a $Z$.
Moving backward in the sequence from this $Z$, we get that the $259^{\text{th}}$ letter is a $Y$, and the $258^{\text{th}}$ letter is an $X$.

ANSWER: (C)

12. The first Wednesday of a month must occur on one of the first 7 days of a month, that is, the $1^{\text{st}}$, $2^{\text{nd}}$, $3^{\text{rd}}$, $4^{\text{th}}$, $5^{\text{th}}$, $6^{\text{th}}$, or $7^{\text{th}}$ of the month.
The second Wednesday of a month occurs 7 days following the first Wednesday of that month, and the third Wednesday of a month occurs 14 days following the first Wednesday of that month.
Adding 14 days to each of the possible dates for the first Wednesday of the month, we get that the third Wednesday of a month must occur on the $15^{\text{th}}$, $16^{\text{th}}$, $17^{\text{th}}$, $18^{\text{th}}$, $19^{\text{th}}$, $20^{\text{th}}$, or $21^{\text{st}}$ of that month.
Of the given answers, the public holiday cannot occur on the $22^{\text{nd}}$ of that month.

ANSWER: (B)

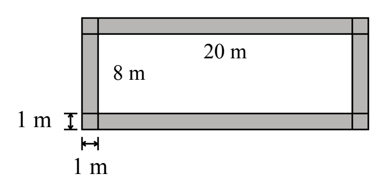
13. The probability that the arrow stops on the largest section is 50% or $\frac{1}{2}$.
The probability that it stops on the next largest section is 1 in 3 or $\frac{1}{3}$.
Thus, the probability that the arrow stops on the smallest section is $1 - \frac{1}{2} - \frac{1}{3} = \frac{6}{6} - \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$.
Answer: (C)

14. A positive number is divisible by both 3 and 4 if it is divisible by 12 (and a positive number that is divisible by 12 is divisible by both 3 and 4).
The positive two-digit numbers that are divisible by 12 (and thus 3 and 4) are $12, 24, 36, 48, 60, 72, 84$, and $96$.
Of these, $60, 72, 84$, and $96$ have a tens digit that is greater than its ones digit.
Thus, there are 4 positive two-digit numbers that satisfy the given property.
Answer: (A)

15. *Solution 1*
The area of the walkway is equal to the area of the pool subtracted from the combined area of the pool and walkway.
That is, if the area of the walkway is $A_w$ and the area of the pool is $A_p$, then $A_w = (A_w + A_p) - A_p$.
The combined area of the pool and walkway is equal to the area of the rectangle with length 22 m and width 10 m.
The length, 22 m, is given by the 20 m pool length plus the 1 m wide walkway on each end of the pool.
Similarly, the 10 m width is given by the 8 m pool width plus the 1 m wide walkway on each side of the pool.
Thus the area of the walkway is

$$A_w = (A_w + A_p) - A_p = (22 \text{ m} \times 10 \text{ m}) - (20 \text{ m} \times 8 \text{ m}) = 220 \text{ m}^2 - 160 \text{ m}^2 = 60 \text{ m}^2$$

*Solution 2*
We begin by extending each side of the pool 1 m in each direction.
This divides the area of the walkway into four 1 m by 1 m squares (in the corners), and two 20 m by 1 m rectangles (along the 20 m sides of the pool), and two 8 m by 1 m rectangles (along the 8 m sides of the pool), as shown.




Thus the area of the walkway is

$$4 \times (1 \text{ m} \times 1 \text{ m}) + 2 \times (20 \text{ m} \times 1 \text{ m}) + 2 \times (8 \text{ m} \times 1 \text{ m}) = 4 \text{ m}^2 + 40 \text{ m}^2 + 16 \text{ m}^2 = 60 \text{ m}^2$$

Answer: (B)

16. Reading from the Venn diagram, 5 students participate in both music and sports, 15 students participate in music (and not sports), and 20 students participate in sports (and not music).
Thus, there are $5 + 15 + 20 = 40$ students that participate in music or sports or both, and so there are $50 - 40 = 10$ students that do not participate in music and do not participate in sports.
Of the 50 students, $\frac{10}{50} \times 100\% = 20\%$ do not participate in music and do not participate in sports.
Answer: (C)

17. If the number of golf balls in Bin G is $x$, then the number of golf balls in Bin F is $\frac{2}{3}x$.
In this case, the total number of golf balls is $x + \frac{2}{3}x = \frac{3}{3}x + \frac{2}{3}x = \frac{5}{3}x$, and so $\frac{5}{3}x = 150$.
Multiplying both sides of this equation by 3, we get $5x = 150 \times 3 = 450$.
Dividing both sides by 5, we get $x = \frac{450}{5} = 90$ and so there are 90 golf balls in Bin G.
The number of golf balls in Bin F is $150 - 90 = 60$, and so there are $90 - 60 = 30$ fewer golf balls in Bin F than in Bin G.
(Alternately, we may have begun by letting there be $3x$ golf balls in Bin G and $2x$ golf balls in Bin F.)

ANSWER: (B)

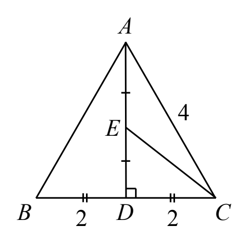
18. Figure 1 is formed with 7 squares.
Figure 2 is formed with $5 + 7$ squares.
Figure 3 is formed with $5 + 5 + 7 = 2 \times 5 + 7$ squares.
Figure 4 will be formed with $5 + 5 + 5 + 7 = 3 \times 5 + 7$ squares.
Figure 5 will be formed with $5 + 5 + 5 + 5 + 7 = 4 \times 5 + 7$ squares.
Thus, the number of groups of 5 squares needed to help form each figure is increasing by 1.
Also, in each case the number of groups of 5 squares needed is one less than the Figure number.
For example, Figure 6 will be formed with 5 groups of 5 squares plus 7 additional squares.
In general, we can say that Figure $N$ will be formed with $N - 1$ groups of 5 squares, plus 7 additional squares.
Since $2022 = 403 \times 5 + 7$, the figure with 2022 squares has 403 groups of 5 squares, plus 7 additional squares.
Thus, $N - 1 = 403$ and so $N = 404$. The number of the figure that has 2022 squares is 404.

ANSWER: (C)

19. There are 60 minutes in an hour, and so the number of minutes between 7 a.m. and 11 a.m. is $4 \times 60 = 240$.
Since Mateo stopped for a 40 minute break, he drove for $240 - 40 = 200$ minutes.
Thus, the average speed for Mateo's 300 km trip was $\dfrac{300 \text{ km}}{200 \text{ minutes}} = 1.5$ km/min.
Since there are 60 minutes in an hour, if Mateo averaged 1.5 km per minute, then he averaged $1.5 \times 60$ km per hour or 90 km/h.

ANSWER: (C)

20. Since $\triangle ABC$ is equilateral and has sides of length 4, then $AB = BC = AC = 4$.
The midpoint of $BC$ is $D$ and so $BD = CD = 2$.
The midpoint of $AD$ is $E$ and so $AE = ED$.
Since $AB = AC$ and $D$ is the midpoint of $BC$, then $AD$ is perpendicular to $BC$, as shown.




Triangle $ADC$ is a right-angled triangle, and so by the Pythagorean Theorem, we get $(AC)^2 = (AD)^2 + (DC)^2$ or $4^2 = (AD)^2 + 2^2$, and so $(AD)^2 = 16 - 4 = 12$.
Similarly, $\triangle EDC$ is right-angled, and so by the Pythagorean Theorem, we get $(EC)^2 = (ED)^2 + (DC)^2$ or $(EC)^2 = (ED)^2 + 2^2$.
Since $ED = \frac{1}{2}AD$, then $(ED)^2 = \frac{1}{2}AD \times \frac{1}{2}AD$ or $(ED)^2 = \frac{1}{4}(AD)^2$.
Since $AD^2 = 12$, then $(ED)^2 = \frac{1}{4} \times 12 = 3$.
Substituting, we get $(EC)^2 = 3 + 2^2$, and so $(EC)^2 = 7$.

ANSWER: (A)

21. A perfect square is a number that can be expressed as the product of two equal integers.
By this definition, 0 is a perfect square since $0 \times 0 = 0$.
Since the product of 0 and every positive integer is 0, then every positive integer is a factor of 0, and so 0 has an infinite number of positive factors.
The next three smallest perfect squares are $1^2 = 1$, $2^2 = 4$ and $3^2 = 9$.
Each of these has at most three positive factors.
The next largest perfect square is $4^2 = 16$.
The positive factors of 16 are 1, 2, 4, 8, and 16, and so 16 is a perfect square that has exactly five positive factors.
The remaining perfect squares that are less than 100 are 25, 36, 49, 64, and 81.
Both 25 and 49 each have exactly three positive factors.
The positive factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.
The positive factors of 64 are 1, 2, 4, 8, 16, 32, and 64.
Thus, both 36 and 64 each have more than five positive factors.
Finally, the positive factors of 81 are 1, 3, 9, 27, and 81.
The two perfect squares that are less than 100 and that have exactly five positive factors are 16 and 81, and their sum is $16 + 81 = 97$.

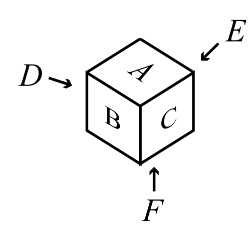
Answer: (E)

22. The sum of the values of each group of three consecutive letters is 35.
Thus, $r + s + t = 35$ and $s + t + u = 35$.
Since each of these equations is equal to 35, then the left sides of the two equations are equal to each other.
That is, $r + s + t = s + t + u$ and since $s + t$ is common to both sides of this equation, then $r = u$.
Since $q + u = 15$ and $r = u$, then $q + r = 15$.
Since $p + q + r = 35$ and $q + r = 15$, then $p = 35 - 15 = 20$.
Finally, we get

$$p + q + r + s + t + u + v = p + (q + r + s) + (t + u + v) = 20 + 35 + 35 = 90$$

Answer: (D)

23. Consider folding the net into the cube and positioning the cube with the face labelled $F$ down (on the bottom), and so the face labelled $A$ is on top and the four remaining vertical faces are as shown.




Beginning at $A$, the ant has 4 choices for which face to visit next.
That is, the ant can walk from $A$ to any of the vertical faces, $B$, $C$, $D$, or $E$.
From each of these vertical faces, the ant can walk to $F$ (the bottom face), or the ant can walk to an adjacent vertical face.
We call these two possibilities Case 1 and Case 2, and for each case we consider the number of possible orders in which the ant can visit the faces.

Case 1: The ant's 2nd move is to $F$, the bottom face

We begin by recognizing that there is only 1 choice for the ant's 2nd move in this case.
From $F$, the ant's 3rd move must be back to a vertical face.
The ant cannot return to the vertical face that it has already visited.
Also, the ant cannot move to the vertical face that is opposite the vertical face it has already

visited. Why?
Consider for example that the ant visits, in order, $A, B, F$.
If the ant walks to $E$ (the face opposite $B$) on its 3rd move, then its 4th move must be to $C$ or to $D$ (since it has visited the other four faces).
However, once at $C$ or $D$, the ant is "trapped" since it has already visited all four adjacent faces, and thus cannot get to the sixth face.
Thus from $F$, the ant's 3rd move must be to a vertical face that is not the face already visited, and is not the face opposite the face already visited, and so there are 2 choices for the next move.
From this vertical face, the ant must walk to another vertical face (since it has already visited $A$ and $F$).
One such adjacent face has already been visited and the other has not, and so the ant's 1 choice is for its 4th move to be to the adjacent vertical face it has not visited.
For example, if the order is $A, B, F, C$, then the ant must move to $E$ next.
Finally, the ant's final move must be to the adjacent vertical face that it has not visited.
Summarizing Case 1, there are 4 choices for the first move from $A$ to one of the vertical faces, 1 choice for the 2nd move (to $F$), 2 choices for the 3rd move, and 1 choice for each of the last two moves, and so there are $4 \times 1 \times 2 \times 1 \times 1 = 8$ possible orders.

Case 2: The ant's 2nd move is to an adjacent vertical face

There are two vertical faces adjacent to each vertical face, and so the ant has 2 choices for its 2nd move.
For its 3rd move, the ant can walk to the bottom face $F$, or the ant can walk to the adjacent face that has not been visited.
We call the first of these Case 2a and the second Case 2b.

Case 2a: The ant's 3rd move is to $F$

We begin by recognizing that there is only 1 choice for the ant's 3rd move in this case.
From $F$, the ant's 4th move must be back to a vertical face.
The ant cannot return to either of the two vertical faces that it has already visited, and so there are 2 choices for the ant's 4th move.
For example, if the order is $A, B, C, F$, the ant's 4th move can be to $D$ or to $E$.
The ant's final move is to the final vertical face and thus there is only 1 choice.
Summarizing Case 2a, there are 4 choices for the first move from $A$ to one of the vertical faces, 2 choices for the 2nd move (to an adjacent vertical face), 1 choice for the 3rd move (to $F$), 2 choices for the 4th move, and 1 choice for the last move, and so there are $4 \times 2 \times 1 \times 2 \times 1 = 16$ possible orders.

Case 2b: The ant's 3rd move is to the adjacent face that has not been visited

We begin by recognizing that there is only 1 choice for the ant's 3rd move in this case.
At this point the ant has visited three vertical faces.
The ant's 4th move can be to $F$ or to the final vertical face.
That is, the ant has 2 choices for its 4th move.
If the ant's 4th move is to $F$, then its last move is to the remaining vertical face.
If the ant's 4th move is to the final vertical face, then its last move is to $F$.
That is, once the ant chooses its 4th move, it has only 1 choice for its final move.
Summarizing Case 2b, there are 4 choices for the first move from $A$ to one of the

vertical faces, 2 choices for the 2nd move (to an adjacent vertical face), 1 choice for the 3rd move (to the adjacent vertical face), 2 choices for the 4th move (to $F$ or to the final vertical face), and 1 choice for the last move, and so there are $4\times2\times1\times2\times1 = 16$ possible orders.

Thus, if the ant starts at $A$ and visits each face exactly once, there are $8+16+16 = 40$ possible orders.

Answer: (E)

24. *Solution 1*

We begin by recognizing that numbers with the given property cannot have two digits that are zero. Can you see why?
Thus, numbers with this property have exactly one zero or they have no zeros.
We consider each of these two cases separately.

Case 1: Suppose the number has exactly one digit that is a zero.
Each of the numbers greater than 100 and less than 999 is a three-digit number and so in this case, the number also has two non-zero digits.
Since one of the digits is equal to the sum of the other two digits and one of the digits is zero, then the two non-zero digits must be equal to one another.
The two non-zero digits can equal any integer from 1 to 9, and thus there are 9 possible values for the non-zero digits.
For each of these 9 possibilities, the zero digit can be the second digit in the number or it can be the third digit (the first digit cannot be zero).
That is, there are 9 possible values for the non-zero digits and 2 ways to arrange the three digits, and thus $9 \times 2 = 18$ numbers of this form satisfy the given property.
For example, numbers of this form are 101 and 110, 202 and 220, and so on.

Case 2: Suppose the number has no digits that are equal to zero.
Let the three digits of the number be $a$, $b$ and $c$, arranged in some order.
Assume that $a$ is the largest digit, and so $a = b + c$.
If $a = b$, then $c = 0$ which contradicts our assumption that no digit is equal to 0.
Similarly, if $a = c$, then $b = 0$ and the same contradiction arises.
Thus $a > b$ and $a > c$.
If $a = 1$, then $b = c = 0$ (since $a > b$ and $a > c$), but then $a \neq b + c$.
Therefore, $a$ is at least 2.
If $a = 2$, then $b = c = 1$ and these are the only possible values for $b$ and $c$ when $a = 2$.
In this case, the 3 ways to arrange these digits give the numbers 112, 121 and 211, each with the desired property.
If $a = 3$, then $b$ and $c$ equal 1 and 2, in some order.
In this case, the 6 ways to arrange these digits give the numbers 123, 132, 213, 231, 312, and 321, each with the desired property.
From these cases, we recognize that if $b = c$, then there are 3 possible arrangements of the digits.
However, if the three digits are different from one another, then there are 3 choices for the first digit, 2 choices for second digit and 1 choice for the third digit, and thus $3 \times 2 \times 1 = 6$ ways to arrange the digits.
We consider all possible values of $a, b, c$ and count the arrangements of these digits in the table below.

| Values for $a$ | Values for $b$ and $c$ with the number of arrangements in brackets [ ] | | | | Total number of arrangements |
|---|---|---|---|---|---|
| 2 | 1,1 [3] | | | | 3 |
| 3 | $1,2$ [6] | | | | 6 |
| 4 | $1,3$ [6] | $2,2$ [3] | | | 9 |
| 5 | $1,4$ [6] | $2,3$ [6] | | | 12 |
| 6 | $1,5$ [6] | $2,4$ [6] | $3,3$ [3] | | 15 |
| 7 | $1,6$ [6] | $2,5$ [6] | $3,4$ [6] | | 18 |
| 8 | $1,7$ [6] | $2,6$ [6] | $3,5$ [6] | $4,4$ [3] | 21 |
| 9 | $1,8$ [6] | $2,7$ [6] | $3,6$ [6] | $4,5$ [6] | 24 |

Thus, there are $18+3+6+9+12+15+18+21+24=126$ numbers that satisfy the given property.

*Solution 2*
We begin by recognizing that numbers with the given property cannot have three equal digits. Can you see why?
Thus, numbers with this property have exactly two digits that are equal or all three digits are different.
We consider each of these two cases separately.

Case 1: Suppose the number has exactly two digits that are equal.
The two equal digits cannot be 0 since then the third digit would also be 0.
If for example the two equal digits are 1s, there are two possibilities for the third digit.
The third digit can be 2 (since $1+1=2$) or the third digit can be 0 (since $1+0=1$).
In the cases for which the third digit is equal to the sum of the two equal digits, the equal digits can be $1,2,3$, or 4 and the third digit is $2,4,6,8$, respectively.
(We note that the equal digits cannot be greater than 4 since their sum is greater than 9.)
For each of these 4 possibilities, there are 3 ways to arrange the digits.
For example, when the equal digits are 1s and the third digit is 2, the numbers 112, 121 and 211 have the desired property.
Thus, there are $4\times 3=12$ such numbers for which the third digit is equal to the sum of the two equal digits.
In the cases for which two of the digits are equal and the third digit is 0, the equal digits can be any integer from 1 to 9 inclusive.
For each of these 9 possibilities, there are 2 ways to arrange the digits.
For example, when the equal digits are 1s and the third digit is 0, the numbers 101 and 110 have the desired property.
Thus, there are $9\times 2=18$ numbers which have two equal digits and the third digit is 0.
In total, there are $12+18=30$ numbers which have two equal digits and satisfy the given property.

Case 2: Suppose all three digits are different from one another.
Let the three digits of the number be $a$, $b$ and $c$, arranged in some order with $a>b>c$.
Since $a$ is the largest digit, then $a=b+c$.
If $a=1$, then $b=c=0$ (since $a>b$ and $a>c$), but this is not possible since $b>c$.
Similarly, if $a=2$, then $b=1$ and $c=0$, however these digits do not satisfy the given property.
Therefore, $a$ is at least 3.
If $a=3$, then $b=2$ and $c=1$.

In this case, the 6 ways to arrange these digits give the numbers 123, 132, 213, 231, 312, 321 and each has the desired property.
We consider all possible values of $a, b, c$ in the table below.

| Values for $a$ | Values for $b, c$ | | | |
|---|---|---|---|---|
| 3 | $2, 1$ | | | |
| 4 | $3, 1$ | | | |
| 5 | $4, 1$ | $3, 2$ | | |
| 6 | $5, 1$ | $4, 2$ | | |
| 7 | $6, 1$ | $5, 2$ | $4, 3$ | |
| 8 | $7, 1$ | $6, 2$ | $5, 3$ | |
| 9 | $8, 1$ | $7, 2$ | $6, 3$ | $5, 4$ |

For each of these 16 possibilities in the table above, there are 6 ways to arrange the three digits, and so there are $16 \times 6 = 96$ such numbers.
Thus, there are a total of $30 + 96 = 126$ numbers that satisfy the given property.

ANSWER: (B)

25. Of the 4200 samples to test, let the number of samples that contain blueberry be $x$.
Since each sample either contains blueberry or it does not, then there are $4200 - x$ samples that do not contain blueberry.
Student A reports correctly on 90% of the $x$ samples that contain blueberry.
Thus, Student A reports $\frac{90}{100}x$ of these samples contain blueberry.
Student A reports correctly on 88% of the $4200 - x$ samples not containing blueberry and thus incorrectly reports that $100\% - 88\% = 12\%$ of these samples contain blueberry.
(That is, when a student is wrong when reporting "no blueberry", it means that there is blueberry.)
Therefore, Student A reports $\frac{12}{100}(4200 - x)$ of these samples contain blueberry.
In total, Student A reports that $\frac{90}{100}x + \frac{12}{100}(4200 - x)$ samples contain blueberry.
Similarly, Student B reports that $\frac{98}{100}x + \frac{14}{100}(4200 - x)$ samples contain blueberry, and Student C reports that $\frac{2m}{100}x + \left(\frac{100-4m}{100}\right)(4200 - x)$ samples contain blueberry.
Student B reports 315 more samples as containing blueberry than Student A, and so

$$\left(\tfrac{98}{100}x + \tfrac{14}{100}(4200 - x)\right) - \left(\tfrac{90}{100}x + \tfrac{12}{100}(4200 - x)\right) = 315$$

Clearing fractions by multiplying by 100, the equation becomes

$$98x + 14(4200 - x) - (90x + 12(4200 - x)) = 31\,500$$

Solving this equation, we get

$$\begin{aligned}
98x + 14(4200 - x) - (90x + 12(4200 - x)) &= 31\,500 \\
98x + 58\,800 - 14x - (90x + 50\,400 - 12x) &= 31\,500 \\
98x + 58\,800 - 14x - 90x - 50\,400 + 12x &= 31\,500 \\
6x + 8400 &= 31\,500 \\
6x &= 23\,100 \\
x &= 3850
\end{aligned}$$

Thus, there are 3850 samples that contain blueberry and $4200 - 3850 = 350$ samples that do not contain blueberry.
Together, the three students report

$$\left(\tfrac{98}{100}(3850) + \tfrac{14}{100}(350)\right) + \left(\tfrac{90}{100}(3850) + \tfrac{12}{100}(350)\right) + \left(\tfrac{2m}{100}(3850) + \left(\tfrac{100-4m}{100}\right)(350)\right)$$

samples as containing blueberry.
Simplifying this expression, we get

$$\begin{aligned} &\left(\tfrac{98}{100}(3850) + \tfrac{14}{100}(350)\right) + \left(\tfrac{90}{100}(3850) + \tfrac{12}{100}(350)\right) + \left(\tfrac{2m}{100}(3850) + \left(\tfrac{100-4m}{100}\right)(350)\right) \\ &= \ 3773 + 49 + 3465 + 42 + 77m + 350 - 14m \\ &= \ 7679 + 63m \end{aligned}$$

For some positive integers $m$, together the three students report $7679 + 63m$ samples as containing blueberry.
If $7679 + 63m$ is greater than 8000, then $63m$ is greater than $8000 - 7679 = 321$ and so $m$ is greater than $\frac{321}{63} \approx 5.09$.
Since $m$ is a positive integer, then $m$ is greater than or equal to 6.
Similarly, if $7679 + 63m$ is less than 9000, then $63m$ is less than $9000 - 7679 = 1321$ and so $m$ is less than $\frac{1321}{63} \approx 20.97$.
Since $m$ is a positive integer, then $m$ is less than or equal to 20.
Thus, we want all integers $m$ from 6 to 20 inclusive for which $7679 + 63m$ is equal to a multiple of 5.
An integer is a multiple of 5 exactly when its units (ones) digit is 0 or 5.
The units digit of the total number of samples, $7679 + 63m$, is determined by the sum of the units digits of 7679, which is 9, and the units digit of the value of $63m$.
Thus, $7679 + 63m$ is a multiple of 5 exactly when the value of $63m$ has a units digit of 1 or 6 (since $9 + 1$ has a units digit of 0 and $9 + 6$ has a units digit of 5).
The value of $63m$ has a units digit of 1 exactly when $m$ has a units digit of 7 (since $3 \times 7$ has a units digit of 1).
The value of $63m$ has a units digit of 6 exactly when $m$ has a units digit of 2 (since $3 \times 2$ has a units digit of 6).
The values of $m$ from 6 to 20 inclusive having a units digit of 7 or 2 are $7, 12$ and 17.
We can confirm that when $m$ is equal to $7, 12$ and 17, the values of $7679 + 63m$ are 8120, 8435 and 8750 respectively, as required.
The sum of all values of $m$ for which the total number of samples that the three students report as containing blueberry is a multiple of 5 between 8000 and 9000 is $7 + 12 + 17 = 36$.

Answer: (B)