The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Tuesday, April 12, 2022
(in North America and South America)

Wednesday, April 13, 2022
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by 
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

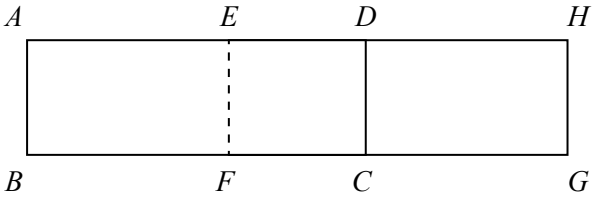
1. In a game, a player throws a ball at a target. If they hit the target, then 7 points are added to their score. If they miss the target, then 3 points are subtracted from their score. A player's score begins at 0, and it is possible for a player to have a negative score.


(a) What is Shane's score after 6 throws if 4 of the throws are hits and 2 of the throws are misses?

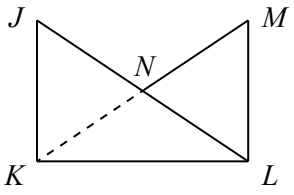

(b) After exactly h hits and 6 misses, Susan's score is 59. What is the value of h?


(c) After exactly 20 throws, Souresh's score is greater than 85 and less than 105. If exactly m of these throws are misses, determine all possible values of the positive integer m.

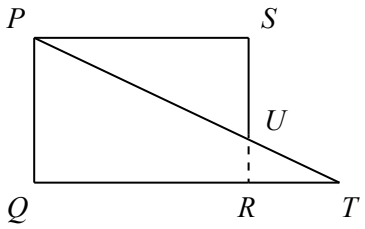
2. (a) Two identical rectangles, $ABCD$ and $EFGH$, each with area 13 cm^2, overlap as shown. The area of the overlapped region, rectangle $EFCD$, is 5 cm^2. What is the area of rectangle $ABGH$?



(b) Two identical right-angled triangles, JKL and MLK, overlap along side KL, as shown. Sides JL and MK intersect at N. The area of the overlapped region, $\triangle KLN$, is equal to half of the area of $\triangle JKL$. The area of the figure $JKLMN$ is 48 cm^2. If $JK = 6$ cm, determine the length of KL.



(c) Rectangle $PQRS$ and $\triangle PQT$ overlap so that R lies on QT, and RS intersects PT at U, as shown. The area of rectangle $PQRS$ is 108 cm^2, and the area of $\triangle PQT$ is 81 cm^2. If the area of the figure $PQTUS$ is 117 cm^2, determine the area of the overlapped region, $PQRU$.



3. If an integer n is written as a product of prime numbers, this product (known as its *prime factorization*) can be used to determine the number of positive factors of n. For example, the prime factorization of $28 = 2 \times 2 \times 7 = 2^2 \times 7^1$. The positive factors of 28 are:

$$28 = 2^2 \times 7^1 \quad 14 = 2^1 \times 7^1 \quad 7 = 2^0 \times 7^1 \quad 4 = 2^2 \times 7^0 \quad 2 = 2^1 \times 7^0 \quad 1 = 2^0 \times 7^0$$

Each positive factor includes 2, 1 or 0 twos, 1 or 0 sevens, and no other prime numbers. Since there are 3 choices for the number of twos, and 2 choices for the number of sevens, there are $3 \times 2 = 6$ positive factors of 28.

(a) How many positive factors does 675 have?

(b) A positive integer n has the positive factors 9, 11, 15, and 25 and exactly fourteen other positive factors. Determine the value of n.

(c) Determine the number of positive integers less than 500 that have the positive factors 2 and 9 and exactly ten other positive factors.

4. Franco and Sarah play a game four times using the following rules:

(R1) The game starts with two jars, each of which might contain some beans.

(R2) Franco goes first, Sarah goes second and they continue to alternate turns.

(R3) On each turn, the player removes a pre-determined number of beans from one of the jars. If neither jar has enough beans in it, the player cannot take their turn and loses. If only one jar has enough beans in it, the player must remove beans from that jar. If both jars have enough beans, the player chooses one of the jars and removes the beans from that jar.

(R4) Franco must attempt to remove 1 bean on his first turn, 3 beans on his second turn, and 4 beans on his third turn. On each of his following sets of three turns, Franco must continue to attempt to remove 1, 3 and 4 beans in sequence.

(R5) Sarah must attempt to remove 2 beans on her first turn and 5 beans on her second turn. On each of her following sets of two turns, Sarah must continue to attempt to remove 2 and 5 beans in sequence.

(R6) A player is declared the winner if the other player loses, as described in (R3).

For example, if the game begins with 10 beans in one jar and 10 beans in the other jar, the sequence of play could be:

Turn Number	1	2	3	4	5	6	7
Number of beans removed by Franco	1		3		4		1
Number of beans removed by Sarah		2		5		2	
Number of beans remaining in the jars	$10, 9$	$10, 7$	$7, 7$	$7, 2$	$3, 2$	$1, 2$	$0, 2$

On the next turn, Sarah cannot remove 5 beans since the greatest number of beans remaining in either jar is 2 and so after exactly 7 turns, Sarah loses and Franco wins.

(a) At the beginning of the first game, there are 40 beans in one jar and 0 beans in the other jar. After a total of 10 turns (5 turns for each of Franco and Sarah), what is the total number of beans left in the two jars?

(b) At the beginning of the second game, there are 384 beans in one jar and 0 beans in the other jar. The game ends with a winner after a total of exactly n turns. What is the value of n?

(c) At the beginning of the third game, there are 17 beans in one jar and 6 beans in the other jar. There is a *winning strategy* that one player can follow to guarantee that they are the winner. Determine which player has a winning strategy and describe this strategy. (A *winning strategy* is a way for a player to choose a jar on each turn so that they win no matter the choices of the other player.)

(d) At the beginning of the fourth game, there are 2023 beans in one jar and 2022 beans in the other jar. Determine which player has a winning strategy and describe this strategy.

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For students...

Thank you for writing the 2022 Fryer Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2022.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2022/2023 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results