The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Wednesday, February 23, 2022
(in North America and South America)

Thursday, February 24, 2022
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $6 + (3 \times 6) - 12$ is

   (A) 6 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

2. The average (mean) of two numbers is 7. One of the numbers is 5. The other number is

   (A) 6 **(B)** 4 **(C)** 3 **(D)** 8 **(E)** 9

3. Gauravi walks every day. One Monday, she walks 500 m. On each day that follows, she increases her distance by 500 m from the previous day. On what day of the week will she walk exactly 4500 m?

   (A) Thursday **(B)** Friday **(C)** Tuesday **(D)** Monday **(E)** Wednesday

4. What is the largest number of squares with side length 2 that can be arranged, without overlapping, inside a square with side length 8?

   (A) 8 **(B)** 32 **(C)** 16 **(D)** 64 **(E)** 4

5. One integer is selected at random from the following list of 15 integers:

   $$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5$$

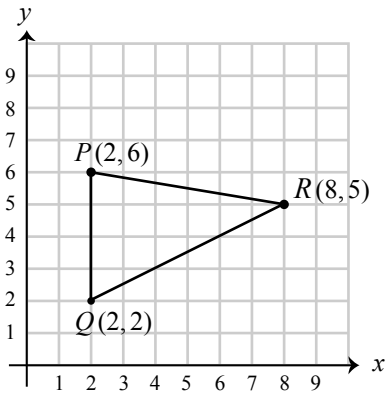
   The probability that the selected integer is equal to n is $\frac{1}{3}$. What is the value of n?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. In the diagram, points $P(2, 6)$, $Q(2, 2)$ and $R(8, 5)$ form a triangle. The area of $\triangle PQR$ is

   (A) 24 **(B)** 14 **(C)** 21

   (D) 12 **(E)** 16



7. The expression $(1 + 2 + 3)(1 + \frac{1}{2} + \frac{1}{3})$ is equal to

   (A) 3 **(B)** 11 **(C)** 6 **(D)** $\frac{11}{6}$ **(E)** 12

8. If $10x + y = 75$ and $10y + x = 57$ for some positive integers x and y, the value of $x + y$ is

(A) 12 **(B)** 5 **(C)** 7 **(D)** 77 **(E)** 132

9. It takes Pearl 7 days to dig 4 holes. It takes Miguel 3 days to dig 2 holes. If they work together and each continues digging at these same rates, how many holes in total will they dig in 21 days?

(A) 35 **(B)** 22 **(C)** 12 **(D)** 26 **(E)** 28

10. If $2^{11} \times 6^5 = 4^x \times 3^y$ for some positive integers x and y, then the value of $x + y$ is

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

Part B: Each correct answer is worth 6.

11. Dhruv is older than Bev. Bev is older than Elcim. Elcim is younger than Andy. Andy is younger than Bev. Bev is younger than Cao. Who is the third oldest?

(A) Andy **(B)** Bev **(C)** Cao **(D)** Dhruv **(E)** Elcim

12. Suppose that d is an odd integer and e is an even integer. How many of the following expressions are equal to an odd integer?

$$d + d \qquad (e + e) \times d \qquad d \times d \qquad d \times (e + d)$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13. Seven identical rectangles are used to create two larger rectangles, as shown in Figure A and Figure B.

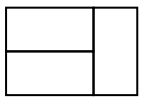


Figure A

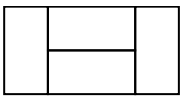


Figure B

The ratio of the perimeter of Figure A to the perimeter of Figure B is

(A) 2 : 3 **(B)** 3 : 4 **(C)** 3 : 5 **(D)** 4 : 5 **(E)** 5 : 6

14. Zebadiah has 3 red shirts, 3 blue shirts, and 3 green shirts in a drawer. Without looking, he randomly pulls shirts from his drawer one at a time. He would like a set of shirts that includes either 3 of the same colour or 3 of different colours. What is the minimum number of shirts that Zebadiah has to pull out to *guarantee* that he has such a set?

(A) 4 **(B)** 3 **(C)** 6 **(D)** 5 **(E)** 7

15. A positive integer a is input into a machine. If a is odd, the output is $a+3$. If a is even, the output is $a+5$. This process can be repeated using each successive output as the next input. For example, if the input is $a=1$ and the machine is used three times, the final output is 12. If the input is $a=15$ and the machine is used 51 times, the final output is

(A) 213 **(B)** 218 **(C)** 212 **(D)** 220 **(E)** 215

16. The remainder when 111 is divided by 10 is 1. The remainder when 111 is divided by the positive integer n is 6. The number of possible values of n is

(A) 5 **(B)** 8 **(C)** 7 **(D)** 6 **(E)** 4

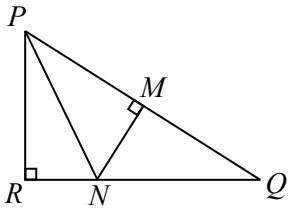
17. An aluminum can in the shape of a cylinder is closed at both ends. Its surface area is 300 cm^2. If the radius of the can were doubled, its surface area would be 900 cm^2. If instead the height of the can were doubled, what would its surface area be?
(The surface area of a cylinder with radius r and height h is equal to $2\pi r^2 + 2\pi rh$.)

(A) 450 cm^2 **(B)** 600 cm^2 **(C)** 750 cm^2 **(D)** 375 cm^2 **(E)** 300 cm^2

18. Aria and Bianca walk at different, but constant speeds. They each begin at 8:00 a.m. from the opposite ends of a road and walk directly toward the other's starting point. They pass each other at 8:42 a.m. Aria arrives at Bianca's starting point at 9:10 a.m. Bianca arrives at Aria's starting point at

(A) 9:30 a.m. **(B)** 9:35 a.m. **(C)** 9:40 a.m. **(D)** 9:45 a.m. **(E)** 9:50 a.m.

19. In the diagram, $\triangle PQR$ is right-angled at R, $PR = 12$, and $QR = 16$. Also, M is the midpoint of PQ and N is the point on QR so that MN is perpendicular to PQ. The area of $\triangle PNR$ is



(A) 21 **(B)** 17.5 **(C)** 36
(D) 16 **(E)** 21.5

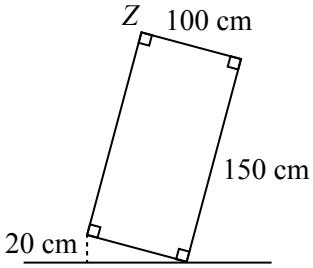
20. A sequence of numbers $t_1, t_2, t_3, \ldots$ has its terms defined by $t_n = \dfrac{1}{n} - \dfrac{1}{n+2}$ for every integer $n \geq 1$. For example, $t_4 = \dfrac{1}{4} - \dfrac{1}{6}$. What is the largest positive integer k for which the sum of the first k terms (that is, $t_1 + t_2 + \cdots + t_{k-1} + t_k$) is less than 1.499?

(A) 2000 **(B)** 1999 **(C)** 2002 **(D)** 2001 **(E)** 1998

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. Gustave has 15 steel bars of masses 1 kg, 2 kg, 3 kg, ..., 14 kg, 15 kg. He also has 3 bags labelled A, B, C. He places two steel bars in each bag so that the total mass in each bag is equal to M kg. How many different values of M are possible?

22. A rectangle with dimensions 100 cm by 150 cm is tilted so that one corner is 20 cm above a horizontal line, as shown. To the nearest centimetre, the height of vertex Z above the horizontal line is $(100 + x)$ cm. What is the value of x?



23. For how many positive integers k do the lines with equations $9x + 4y = 600$ and $kx - 4y = 24$ intersect at a point whose coordinates are positive integers?

24. There are functions $f(x)$ with the following properties:
    - $f(x) = ax^2 + bx + c$ for some integers a, b and c with $a > 0$, and
    - $f(p) = f(q) = 17$ and $f(p+q) = 47$ for some prime numbers p and q with $p < q$.

    For each such function, the value of $f(pq)$ is calculated. The sum of all possible values of $f(pq)$ is S. What are the rightmost two digits of S?

25. In the 3×3 grid shown, the central square contains the integer 5. The remaining eight squares contain a, b, c, d, e, f, g, h, which are each to be replaced with an integer from 1 to 9, inclusive. Integers can be repeated. There are N ways to complete the grid so that the sums of the integers along each row, along each column, and along the two main diagonals are all divisible by 5. What are the rightmost two digits of N?

a	b	c
d	5	e
f	g	h

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For students...

Thank you for writing the 2022 Fermat Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

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- Free copies of past contests
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- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

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