



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Wednesday, February 23, 2022
(in North America and South America)

Thursday, February 24, 2022
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

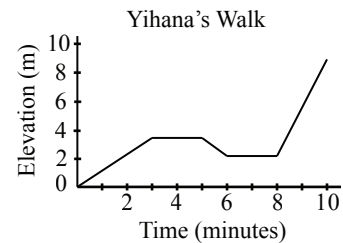
- The expression $2 + (0 \times 2^2)$ is equal to
 (A) 1 (B) 2 (C) 4 (D) 5 (E) 6
- The integer 119 is a multiple of
 (A) 2 (B) 3 (C) 5 (D) 7 (E) 11
- Which of the following fractions has the greatest value?
 (A) $\frac{3}{10}$ (B) $\frac{4}{7}$ (C) $\frac{5}{23}$ (D) $\frac{2}{3}$ (E) $\frac{1}{2}$

- The pattern of shapes $\triangle, \square, \heartsuit, \diamond, \bigcirc$ is repeated to form the sequence

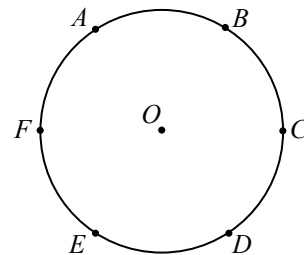
$\triangle, \square, \heartsuit, \diamond, \bigcirc, \triangle, \square, \heartsuit, \diamond, \bigcirc, \dots$

The 22nd shape in the sequence is

- (A) \square (B) \bigcirc (C) \diamond (D) \triangle (E) \heartsuit
- The expression $(5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5)$ is equal to
 (A) 5 (B) 10 000 (C) 625 (D) 19 525 (E) 125
- Yihana walks for 10 minutes. A graph of her elevation in metres versus time in minutes is shown. The length of time for which she was walking uphill is
 (A) 5 minutes (B) 6 minutes (C) 4 minutes
 (D) 7 minutes (E) 8 minutes



- Points $A, B, C, D, E,$ and F are evenly spaced around the circle with centre O , as shown. The measure of $\angle AOC$ is
 (A) 90° (B) 150° (C) 144°
 (D) 120° (E) 108°



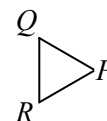
- A rectangle has positive integer side lengths and an area of 24. The perimeter of the rectangle *cannot* be
 (A) 20 (B) 22 (C) 28 (D) 50 (E) 36
- The operation $a \nabla b$ is defined by $a \nabla b = \frac{a+b}{a-b}$ for all integers a and b with $a \neq b$.
 For example, $2 \nabla 3 = \frac{2+3}{2-3} = -5$. If $3 \nabla b = -4$, what is the value of b ?
 (A) 5 (B) -7 (C) 7 (D) -5 (E) 3

10. If x is 20% of y and x is 50% of z , then what percentage is z of y ?
- (A) 70% (B) 10% (C) 30% (D) 40% (E) 60%

Part B: Each correct answer is worth 6.

11. A store sells jellybeans at a fixed price per gram. The price for 250 g of jellybeans is \$7.50. What mass of jellybeans sells for \$1.80?
- (A) 6 g (B) 54 g (C) 60 g (D) 120 g (E) 190 g

12. An equilateral triangle is made of cardboard and lies on a table. Paola stands in front of the table and sees the triangle in the position shown. She flips the triangle over, keeping edge QR on the table throughout the flip. From this position, Paola then flips the triangle again, this time keeping edge PR on the table throughout the flip. What is the resulting position of the triangle that Paola sees?



- (A) (B) (C) (D) (E)

13. Two identical smaller cubes are stacked next to a larger cube. Each of the two smaller cubes has a volume of 8. The combined height of the smaller cubes equals the height of the larger cube. What is the volume of the larger cube?

- (A) 16 (B) 32 (C) 125 (D) 48 (E) 64

14. The integer 48 178 includes the block of digits 178. The three integers 51 870, 19 728 and 38 717 do not include the block of digits 178. How many integers between 10 000 and 100 000 include the block of digits 178?

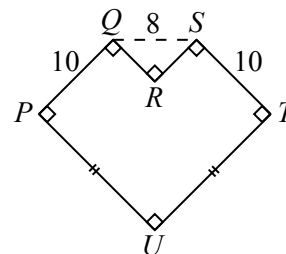
- (A) 280 (B) 300 (C) 270 (D) 310 (E) 260

15. The integers a , b and c satisfy the equations $a + 5 = b$ and $5 + b = c$ and $b + c = a$. The value of b is

- (A) -30 (B) -20 (C) -10 (D) 0 (E) 5

16. In the diagram, hexagon $PQRSTU$ has interior right angles at P , Q , S , T , and U and an exterior right angle at R . Also, $PU = UT$, $PQ = ST = 10$, and $QS = 8$. The perimeter of $PQRSTU$ is closest to

- (A) 48 (B) 56 (C) 63
(D) 71 (E) 72



17. Zebadiah has 3 red shirts, 3 blue shirts, and 3 green shirts in a drawer. Without looking, he randomly pulls shirts from his drawer one at a time. He would like a set of shirts that includes either 3 of the same colour or 3 of different colours. What is the minimum number of shirts that Zebadiah has to pull out to *guarantee* that he has such a set?
- (A) 4 (B) 3 (C) 6 (D) 5 (E) 7
18. At the beginning of the first day, a box contains 1 black ball, 1 gold ball, and no other balls. At the end of each day, for each gold ball in the box, 2 black balls and 1 gold ball are added to the box; this means that at the end of the first day, there are 5 balls in the box. If no balls are removed from the box, how many balls are in the box at the end of the seventh day?
- (A) 395 (B) 371 (C) 389 (D) 377 (E) 383
19. The area of the triangular region bounded by the x -axis, the y -axis and the line with equation $y = 2x - 6$ is one-quarter of the area of the triangular region bounded by the x -axis, the line with equation $y = 2x - 6$ and the line with equation $x = d$, where $d > 0$. What is the value of d ?
- (A) 9 (B) 6 (C) 8 (D) $3 + 3\sqrt{2}$ (E) 15
20. If m and n are positive integers that satisfy the equation $3m^3 = 5n^5$, the smallest possible value for $m + n$ is
- (A) 900 (B) 450 (C) 720 (D) 810 (E) 630

Part C: Each correct answer is worth 8.

Each correct answer is an integer from 0 to 99, inclusive.

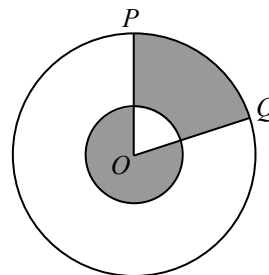
A one-digit answer (such as “7”) must be coded with a leading zero (“07”).

Note: The integer formed by the rightmost two digits of 12 345 is 45.

The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. There are exactly four ordered pairs of positive integers (x, y) that satisfy the equation $20x + 11y = 881$. Mehdi writes down the four values of y and adds the smallest and largest of these values. What is this sum?

22. In the diagram, two circles are centred at O . The smaller circle has a radius of 1 and the larger circle has a radius of 3. Points P and Q are placed on the larger circle so that the areas of the two shaded regions are equal. If $\angle POQ = x^\circ$, what is the value of x ?



23. Andreas, Boyu, Callista, and Diane each randomly choose an integer from 1 to 9, inclusive. Each of their choices is independent of the other integers chosen and the same integer can be chosen by more than one person. The probability that the sum of their four integers is even is equal to $\frac{N}{6561}$ for some positive integer N . What is the sum of the squares of the digits of N ?
24. A cube with edge length 8 is balanced on one of its vertices on a horizontal table such that the diagonal from this vertex through the interior of the cube to the farthest vertex is vertical. When the sun is directly above the top vertex, the shadow of the cube on the table is a regular hexagon. The area of this shadow can be written in the form $a\sqrt{b}$, where a and b are positive integers and b is not divisible by any perfect square larger than 1. What is the value of $a + b$?
25. There are T tokens arranged in a circle for some positive integer T . Moving clockwise around the circle, the tokens are labelled, in order, with the integers from 1 to T . Starting from the token labelled 1, Évariste:
- Removes the token at the current position.
 - Moves clockwise to the next remaining token.
 - Moves clockwise again to the next remaining token.
 - Repeats steps (i) to (iii) until only one token remains.

When $T = 337$, the number on the last remaining token is L . There are other integers T for which the number on the last remaining token is also L . What are the rightmost two digits of the smallest possible value of T ?



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For students...

Thank you for writing the 2022 Cayley Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website cemc.uwaterloo.ca to find

- More information about the Galois Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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