The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Wednesday, November 16, 2022
(in North America and South America)

Thursday, November 17, 2022
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Do not open this booklet until instructed to do so.
There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Write your name, school name, and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

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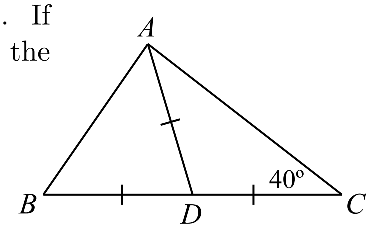
NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. The area of one face of a cube is 16 cm^2. The volume of the same cube is V cm^3. What is the value of V?

2. In the diagram, point D is on side BC of $\triangle ABC$. If $BD = CD = AD$ and $\angle ACD = 40^\circ$, what is the measure of $\angle BAC$?



3. Marie-Pascale solves 4 math problems per day. Kaeli solves x math problems per day. Over a number of days, Marie-Pascale solves a total of 72 problems. Over the same number of days, Kaeli solves 54 more problems than Marie-Pascale. What is the value of x?

4. If a, b, c, and d satisfy $\frac{a}{b} = \frac{2}{3}$ and $\frac{c}{b} = \frac{1}{5}$ and $\frac{c}{d} = \frac{7}{15}$, what is the value of $\frac{ab}{cd}$?

5. Magnus and Viswanathan play a game against each other three times:
   - In each game, each player's score is a positive integer. The players' scores at the end of the three games are six different integers.
   - In each game, the winner's score is at least 25 points. If the winner's score is 25, their opponent's score must be at most 23. If the winner's score is greater than 25, their opponent's score must be exactly 2 less than the winner's score.
   - Viswanathan wins either the first game or the second game, but not both.
   - Viswanathan wins the third game with a score of 25.
   - Each player's score in the second game is the average of their scores in the first and third games.

   What was Magnus's score in the third game?

6. How many ways are there to choose integers a, b and c with $a < b < c$ from the list 1, 5, 8, 21, 22, 27, 30, 33, 37, 39, 46, 50 so that the product abc is a multiple of 12?

PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. In each part of this problem, seven different positive integers will be placed in the seven boxes of the "H"-shaped figure. The integers must be placed so that the three integers in the left vertical column, the three integers in the right vertical column, and the three integers in the one horizontal row all have the same sum. For example, in the "H" to the right, $8+11+12 = 31$ and $11+19+1 = 31$ and $27+1+3 = 31$.

8		27
11	19	1
12		3

   (a) Place the integers 3, 5, 7, 15 in the figure so that each of the three sums is equal to 29.

13		11
9		

   (b) There is a value of t for which the figure shown has three equal sums and contains seven different integers. Determine this value of t.

15		$2t - 3$
$t + 1$	16	10
11		14

   (c) Seven different positive integers are placed in the figure so that the three sums are equal. If $a < c$, determine all possible values of a.

a		b
12	9	7
c		11

   (d) The integers k and n are each between 4 and 18, inclusive. The figure contains seven different integers and the three sums are equal. Determine all possible values of k.

7		$n + 6$
10	n	18
k		4

2. A line that is neither horizontal nor vertical intersects the y-axis when $x = 0$ and intersects the x-axis when $y = 0$. For example, the line with equation $4x + 5y = 40$ intersects the y-axis at $(0, 8)$ (because $4 \times 0 + 5y = 40$ gives $y = 8$) and the x-axis at $(10, 0)$ (because $4x + 5 \times 0 = 40$ gives $x = 10$).

   (a) The line with equation $2x + 3y = 12$ intersects the y-axis at A and the x-axis at B. If O is the origin, determine the area of $\triangle AOB$.

   (b) Suppose that $c > 0$. The line with equation $6x + 5y = c$ intersects the y-axis at D and the x-axis at E. If O is the origin and the area of $\triangle DOE$ is 240, determine the value of c.

   (c) Suppose that m and n are integers with $100 \leq m$ and $m < n$. The line with equation $(2m)x + y = 4m$ intersects the y-axis at P and the x-axis at Q. The line with equation $(7n)x + 4y = 28n$ intersects the y-axis at S and the x-axis at R. If quadrilateral $PQRS$ has area 2022, determine two possible pairs (m, n).

3. A straight path is 2 km in length. Beatrice walks from the beginning of the path to the end of the path at a constant speed of 5 km/h. Hieu cycles from the beginning of the path to the end of the path at a constant speed of 15 km/h.

   (a) Suppose that Hieu starts 10 minutes later than Beatrice. Determine the number of minutes that it takes Hieu to catch up to Beatrice on the path.

   (b) Suppose that Beatrice starts at a time that is exactly b minutes after 9:00 a.m. and that Hieu starts at a time that is exactly h minutes after 9:00 a.m., where b and h are integers from 0 to 59, inclusive, that are each randomly and independently chosen.

      (i) Determine the probability that there is a time at which Beatrice and Hieu are at the same place on the path. (Beatrice and Hieu are at the same place at a given time if they start at the same time or finish at the same time, or are at the same place at some point between the beginning and end of the path at the same time.)

      (ii) One day, Beatrice uses a scooter and travels from the beginning of the path to the end of the path at a constant speed of x km/h, where $x > 5$ and $x < 15$. Hieu still cycles from the beginning of the path to the end of the path at a constant speed of 15 km/h. If the probability that there is a time at which Beatrice and Hieu are at the same place on the path is $\frac{13}{200}$, determine the range of possible values of x.