The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Galois Contest

(Grade 10)

April 2021
(in North America and South America)

April 2021
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 75 minutes   ©2021 University of Waterloo

Do not open this booklet until instructed to do so.

Number of questions: 4   **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 💡

   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by ✐

   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

   - Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
   - Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

1. The operation $\triangle$ is defined by $a \triangle b = a(2b + 4)$ for integers a and b. For example, $3 \triangle 6 = 3(2 \times 6 + 4) = 3(16) = 48$.

   (a) What is the value of $5 \triangle 1$?

   (b) If $k \triangle 2 = 24$, what is the value of k?

   (c) Determine all values of p for which $p \triangle 3 = 3 \triangle p$.

   (d) Determine all values of m for which $m \triangle (m + 1) = 0$.

2. The organizer for a sports league with four teams has entered some of the end-of-season data into the table shown. Each team played 27 games and each game resulted in a win for one team and a loss for the other team, or in a tie for both teams. Each team earned 2 points for a win, 0 points for a loss, and 1 point for a tie.

Team Name	Games Played	Number of Wins	Number of Losses	Number of Ties	Total Points
P	27	10	14		23
Q	27				
R	27				25
S	27				

(a) How many ties did Team P have at the end of the season?

(b) Team Q had 2 more wins than Team P and 4 fewer losses than Team P. How many total points did Team Q have at the end of the season?

(c) Explain why Team R could not have finished the season with exactly 6 ties.

(d) At the end of the season, Team S had 4 more wins than losses. Show that Team S must have finished the season with a total of 31 points.

3. Rectangle $ABCD$ has vertices $A(0,0)$, $B(0,12)$, $C(6,12)$, and $D(6,0)$.

(a) Diagonals AC and BD intersect at point E. What is the area of $\triangle ADE$?

(b) Point $P(0,p)$ lies on line segment AB. The area of trapezoid $BCDP$ is twice the area of $\triangle PAD$. What is the value of p?

(c) The line passing through $U(0,u)$, $V(2,4)$ and $W(6,w)$ divides $ABCD$ into two trapezoids. Determine all possible pairs of points U and W for which the ratio of the areas of these two trapezoids is $5 : 3$.

4. (a) If $\dfrac{5}{x} + \dfrac{14}{y} = 2$ and $x = 6$, what is the value of y?

(b) Determine all possible ordered pairs of positive integers (x, y) that are solutions to the equation $\dfrac{4}{x} + \dfrac{5}{y} = 1$.

(c) Consider the equation $\dfrac{16}{x} + \dfrac{25}{y} = p$, where p is a prime number and $p \geq 5$.

Determine all possible values of p for which there is at least one ordered pair of positive integers (x, y) that is a solution to the equation.

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For students...

Thank you for writing the 2021 Galois Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2021.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2021/2022 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for high school students
- Learn about our face-to-face workshops and our web resources
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- Find your school's contest results