The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

April 2021
(in North America and South America)

April 2021
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. A company sells rectangular business cards. Each card has dimensions 5 cm $\times$ 9 cm. Cards are printed on a page and then the page is cut to produce the individual cards.

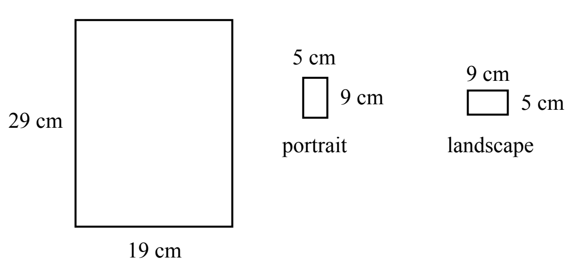
   (a) What is the area of each business card in cm^2?

   (b) Several business cards are printed without overlapping on a single 20 cm $\times$ 27 cm page. If the entire page is used with no waste, how many business cards are printed?

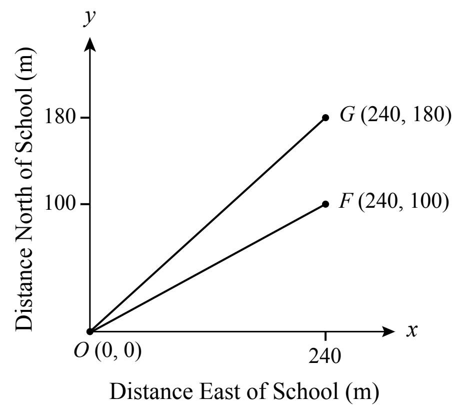
   (c) Several business cards are to be printed on 19 cm $\times$ 29 cm pages in one of two possible ways:

   - The *portrait* page layout is printed so that every card is positioned with its 5 cm edges parallel to the 19 cm edges of the page.
   - The *landscape* page layout is printed so that every card is positioned with its 5 cm edges parallel to the 29 cm edges of the page.

   Which of these two page layouts allows the greatest number of business cards from a single page?



2. Franklin and Giizhig travel from their school to their own homes each day. The school is located at $O(0, 0)$. Franklin's home is at $F(240, 100)$ and Giizhig's home is at $G(240, 180)$. The straight paths from their school to each of their homes are shown on the graph. (Throughout this problem, all coordinates represent lengths in metres.)



(a) What is the distance, in metres, along the straight path from the school to Franklin's home?

(b) On Monday, Franklin walks at a constant speed of 80 m/min. How many minutes does it take Franklin to walk from school straight to his home?

(c) On Tuesday, Franklin and Giizhig leave school at the same time. Franklin walks at 80 m/min straight to his own home and then immediately turns and walks straight toward Giizhig's home. Giizhig walks at g m/min straight to her own home and then immediately turns and walks straight toward Franklin's home. If they meet exactly halfway between their homes, what is the value of g?

3. Given a list of six numbers, the *Reverse Operation*, R_n, reverses the order of the first n numbers in the list, where n is an integer and $2 \leq n \leq 6$. For example, if the list is $1, 4, 6, 2, 3, 5$, then after performing R_4 the list becomes $2, 6, 4, 1, 3, 5$.

(a) R_3 is performed on the list $5, 2, 3, 1, 4, 6$. What is the new list?

(b) A Reverse Operation is performed on the list $1, 2, 3, 4, 5, 6$. A second Reverse Operation is performed on the resulting list to give the final list $3, 4, 2, 1, 5, 6$. Which two Reverse Operations were performed and in what order were they performed?

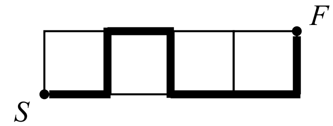
(c) Suppose that m is the minimum number of Reverse Operations that need to be performed, in order, on the list $1, 2, 3, 4, 5, 6$ so that 3 ends up in the last position (that is, the list takes the form $\square, \square, \square, \square, \square, 3$). The value of m can be determined by answering (i) and (ii), below.

  (i) Find m Reverse Operations and show that after performing them, the desired result is achieved (that 3 ends up in the last position).

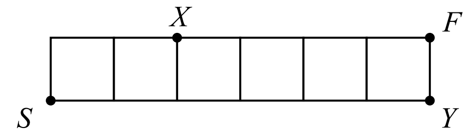
  (ii) Explain why performing fewer than m Reverse Operations can never achieve the desired result.

(d) Determine the minimum number of Reverse Operations that need to be performed, in order, on the list $1, 2, 3, 4, 5, 6$ so that the last number in the list is 4 and the second last number in the list is 5 (that is, the list takes the form $\square, \square, \square, \square, 5, 4$).

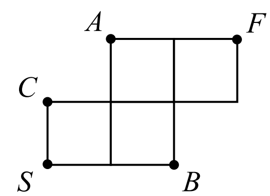
4. An *SF path* starts at S, follows along the edges of the squares, never visits any vertex more than once, and finishes at F. An example of an SF path is shown. (A vertex is a point where two or more of the squares' edges meet.)



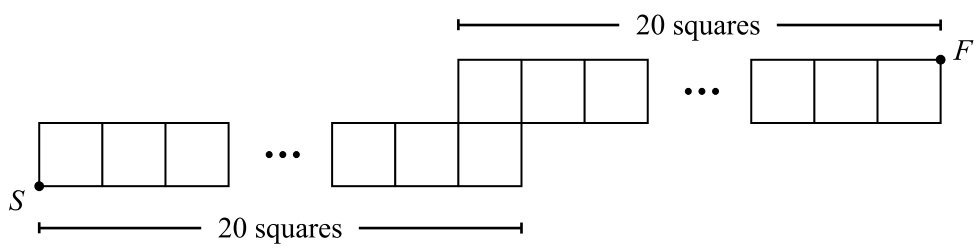
(a) In your solution booklet, draw the SF path that passes through each vertex *except* X and Y.



(b) Explain why no SF path passes through all three of the vertices A, B and C in the diagram shown.



(c) Determine the number of SF paths in the diagram below.



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Thank you for writing the 2021 Fryer Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2021.

Visit our website `cemc.uwaterloo.ca`