The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Cayley Contest

(Grade 10)

Tuesday, February 23, 2021
(in North America and South America)

Wednesday, February 24, 2021
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $\dfrac{2+4}{1+2}$ is equal to

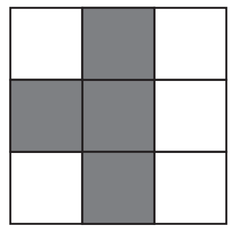
   (A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

2. The ones (units) digit of 542 is 2. When 542 is multiplied by 3, the ones (units) digit of the result is

   (A) 9 **(B)** 3 **(C)** 5 **(D)** 4 **(E)** 6

3. Some of the 1×1 squares in a 3×3 grid are shaded, as shown. What is the perimeter of the shaded region?

   (A) 10 **(B)** 14 **(C)** 8 **(D)** 18 **(E)** 20



4. If $3x + 4 = x + 2$, the value of x is

   (A) 0 **(B)** -4 **(C)** -3 **(D)** -1 **(E)** -2

5. Which of the following is equal to 110% of 500?

   (A) 610 **(B)** 510 **(C)** 650 **(D)** 505 **(E)** 550

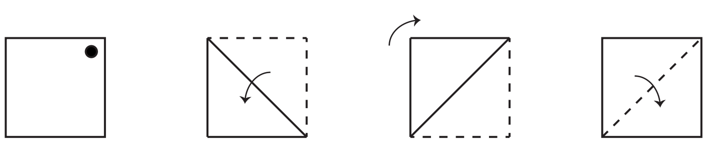
6. Eugene swam on Sunday, Monday and Tuesday. On Monday, he swam for 30 minutes. On Tuesday, he swam for 45 minutes. His average swim time over the three days was 34 minutes. For how many minutes did he swim on Sunday?

   (A) 20 **(B)** 25 **(C)** 27 **(D)** 32 **(E)** 37.5

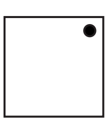
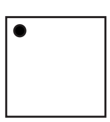
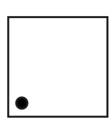
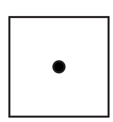
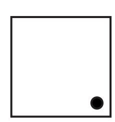
7. For which of the following values of x is $x^3 < x^2$?

   (A) $x = \frac{5}{3}$ **(B)** $x = \frac{3}{4}$ **(C)** $x = 1$ **(D)** $x = \frac{3}{2}$ **(E)** $x = \frac{21}{20}$

8. A square piece of paper has a dot in its top right corner and is lying on a table. The square is folded along its diagonal, then rotated 90° clockwise about its centre, and then finally unfolded, as shown.

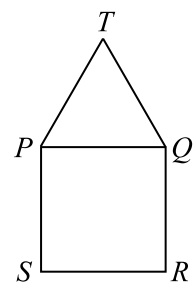


   The resulting figure is

   (A)  **(B)**  **(C)**  **(D)**  **(E)** 

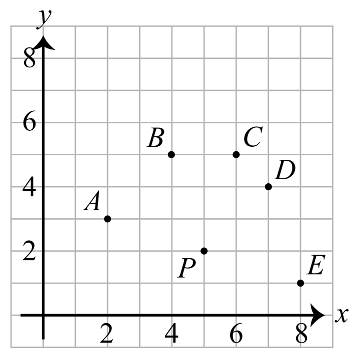
9. In 12 years, Janice will be 8 times as old as she was 2 years ago. How old is Janice now?
(A) 4 **(B)** 8 **(C)** 10 **(D)** 2 **(E)** 6

10. In the diagram, pentagon $TPSRQ$ is constructed from equilateral $\triangle PTQ$ and square $PQRS$. The measure of $\angle STR$ is equal to
(A) 10° **(B)** 15° **(C)** 20°
(D) 30° **(E)** 45°



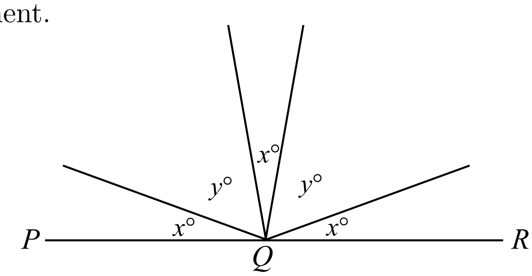
Part B: Each correct answer is worth 6.

11. In the diagram, which of the following points is at a different distance from P than the rest of the points?
(A) A **(B)** B **(C)** C
(D) D **(E)** E



12. If $x = 2$ and $y = x^2 - 5$ and $z = y^2 - 5$, then z equals
(A) -6 **(B)** -8 **(C)** 4 **(D)** 76 **(E)** -4

13. In the diagram, PQR is a straight line segment. If $x + y = 76$, what is the value of x?
(A) 28 **(B)** 30 **(C)** 35
(D) 36 **(E)** 38



14. The line with equation $y = 2x - 6$ is reflected in the y-axis. What is the x-intercept of the resulting line?
(A) -12 **(B)** 6 **(C)** -6 **(D)** -3 **(E)** 0

15. Amy bought and then sold $15n$ avocados, for some positive integer n. She made a profit of \$100. (Her *profit* is the difference between the total amount that she earned by selling the avocados and the total amount that she spent in buying the avocados.) She paid \$2 for every 3 avocados. She sold every 5 avocados for \$4. What is the value of n?
(A) 100 **(B)** 20 **(C)** 50 **(D)** 30 **(E)** 8

16. If $3^x = 5$, the value of 3^{x+2} is
(A) 10 **(B)** 25 **(C)** 2187 **(D)** 14 **(E)** 45

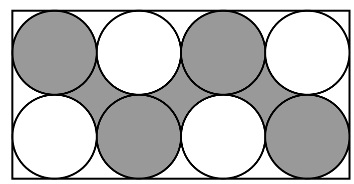
17. A group of friends are sharing a bag of candy.
On the first day, they eat $\frac{1}{2}$ of the candies in the bag.
On the second day, they eat $\frac{2}{3}$ of the remaining candies.
On the third day, they eat $\frac{3}{4}$ of the remaining candies.
On the fourth day, they eat $\frac{4}{5}$ of the remaining candies.
On the fifth day, they eat $\frac{5}{6}$ of the remaining candies.
At the end of the fifth day, there is 1 candy remaining in the bag.
How many candies were in the bag before the first day?

(A) 512 **(B)** 720 **(C)** 1024 **(D)** 1440 **(E)** 2048

18. Elina and Gustavo leave Cayley H.S. at 3:00 p.m. Elina runs north at a constant speed of 12 km/h. Gustavo walks east at a constant speed of 5 km/h. After 12 minutes, Elina and Gustavo change direction and travel directly towards each other, still at 12 km/h and 5 km/h, respectively. The time that they will meet again is closest to

(A) 3:24 p.m. **(B)** 3:35 p.m. **(C)** 3:25 p.m. **(D)** 3:29 p.m. **(E)** 3:21 p.m.

19. In the diagram, eight circles, each of radius 1, are drawn inside a rectangle. Four of the circles are tangent to two sides of the rectangle and to two other circles. Four of the circles are tangent to one side of the rectangle and to three other circles. A region has been shaded, as shown. It consists of three spaces (each space bounded by a different set of four circles), as well as four of the circles themselves. The area of this region is closest to



(A) 12 **(B)** 13 **(C)** 14
(D) 15 **(E)** 16

20. How many four-digit positive integers are divisible by both 12 and 20, but are not divisible by 16?

(A) 111 **(B)** 113 **(C)** 125 **(D)** 150 **(E)** 149

Part C: Each correct answer is worth 8.

21. The variables a, b, c, d, e, and f represent the numbers 4, 12, 15, 27, 31, and 39 in some order. Suppose that

$$a + b = c$$
$$b + c = d$$
$$c + e = f$$

The value of $a + c + f$ is

(A) 58 **(B)** 70 **(C)** 73 **(D)** 82 **(E)** 85

22. The cells of a 3×3 grid are to be filled with integers so that the average value of the entries along each row, each column, and each diagonal is the same. The integers 10, 64 and 70 are entered, as shown. When the remaining six squares are filled in to complete the grid, what integer replaces x?

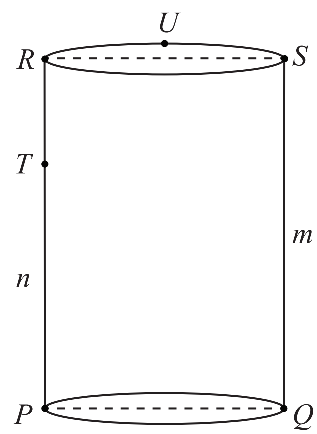
64	10	
70		
	x	

(A) 78 **(B)** 82 **(C)** 86
(D) 90 **(E)** 94

23. A special six-sided die has its faces numbered 1 through 6 and has the property that rolling each number x is x times as likely as rolling a 1. For example, the probability of rolling a 5 is 5 times the probability of rolling a 1, while the probability of rolling a 2 is 2 times the probability of rolling a 1. Robbie and Francine play a game where they each roll this die three times, and the total of their three rolls is their score. The winner is the player with the highest score; if the two players are tied, neither player wins. After two rolls each, Robbie has a score of 8 and Francine has a score of 10. The probability that Robbie will win can be written in lowest terms as $\dfrac{r}{400+s}$, where r and s are positive integers. What is value of $r+s$?

(A) 96 **(B)** 86 **(C)** 76 **(D)** 66 **(E)** 56

24. In the diagram, PQ is a diameter of the circular base of the cylinder. RS is a diameter of the top face of the cylinder and is directly above PQ, as shown. Point U is on the circumference of the top face, halfway between R and S. Point T is on the cylinder and is directly above P. Suppose that $QS = m$ and $PT = n$, where m and n are integers with $1 < n < m$. If $QU = 9\sqrt{33}$ and $UT = 40$, what is the remainder when the integer equal to QT^2 is divided by 100?



(A) 29 **(B)** 49 **(C)** 9
(D) 89 **(E)** 69

25. The points $J(2,7)$, $K(5,3)$ and $L(r,t)$ form a triangle whose area is less than or equal to 10. Let $\mathcal{R}$ be the region formed by all such points L with $0 \leq r \leq 10$ and $0 \leq t \leq 10$. When written as a fraction in lowest terms, the area of $\mathcal{R}$ is equal to $\dfrac{300+a}{40-b}$ for some positive integers a and b. The value of $a+b$ is

(A) 82 **(B)** 71 **(C)** 60 **(D)** 49 **(E)** 93

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For students...

Thank you for writing the 2021 Cayley Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Galois Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

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Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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- Find your school's contest results