The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Pascal Contest

(Grade 9)

Tuesday, February 25, 2020

(in North America and South America)

Wednesday, February 26, 2020

(outside of North America and South America)

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

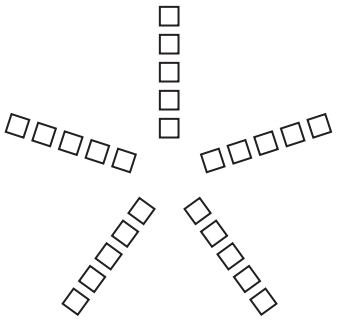
Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. How many □ symbols are in the figure?



(A) 24 **(B)** 20 **(C)** 15
(D) 17 **(E)** 25

2. The value of $0.8 + 0.02$ is

(A) 0.28 **(B)** 8.02 **(C)** 0.82 **(D)** 0.16 **(E)** 0.01

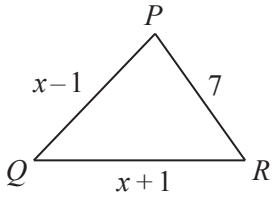
3. If $2x + 6 = 16$, the value of $x + 4$ is

(A) 7 **(B)** 8 **(C)** 9 **(D)** 15 **(E)** 13

4. When two positive integers are multiplied, the result is 24. When these two integers are added, the result is 11. When the smaller integer is subtracted from the larger integer, the result is

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. In the diagram, $\triangle PQR$ has side lengths as shown. If $x = 10$, the perimeter of $\triangle PQR$ is



(A) 29 **(B)** 31 **(C)** 25
(D) 27 **(E)** 23

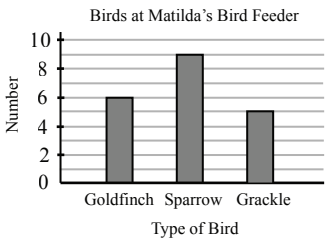
6. The value of $\dfrac{2^4 - 2}{2^3 - 1}$ is

(A) 1 **(B)** 0 **(C)** $\frac{7}{4}$ **(D)** $\frac{4}{3}$ **(E)** 2

7. Ewan writes out a sequence where he counts by 11s starting at 3. The resulting sequence is $3, 14, 25, 36, \ldots$. A number that will appear in Ewan's sequence is

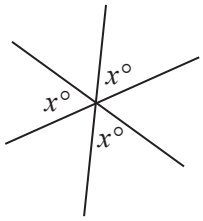
(A) 113 **(B)** 111 **(C)** 112 **(D)** 110 **(E)** 114

8. Matilda counted the birds that visited her bird feeder yesterday. She summarized the data in the bar graph shown. The percentage of birds that were goldfinches is



(A) 15% **(B)** 20% **(C)** 30%
(D) 45% **(E)** 60%

9. In the diagram, three lines intersect at a point. What is the value of x?



(A) 30 **(B)** 45 **(C)** 60
(D) 90 **(E)** 120

10. Starting at 1:00 p.m., Jorge watched three movies. The first movie was 2 hours and 20 minutes long. He took a 20 minute break and then watched the second movie, which was 1 hour and 45 minutes long. He again took a 20 minute break and then watched the last movie, which was 2 hours and 10 minutes long. At what time did the final movie end?

(A) 6:45 p.m. **(B)** 7:15 p.m. **(C)** 7:35 p.m. **(D)** 7:55 p.m. **(E)** 8:15 p.m.

Part B: Each correct answer is worth 6.

11. Anna thinks of an integer.

- It is *not* a multiple of three.
- It is *not* a perfect square.
- The sum of its digits is a prime number.

The integer that Anna is thinking of could be

(A) 12 **(B)** 14 **(C)** 16 **(D)** 21 **(E)** 26

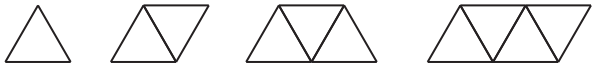
12. Natalie and Harpreet are the same height. Jiayin's height is 161 cm. The average (mean) of the heights of Natalie, Harpreet and Jiayin is 171 cm. What is Natalie's height?

(A) 161 cm **(B)** 166 cm **(C)** 176 cm **(D)** 183 cm **(E)** 191 cm

13. The ratio of apples to bananas in a box is $3 : 2$. The total number of apples and bananas in the box *cannot* be equal to

(A) 40 **(B)** 175 **(C)** 55 **(D)** 160 **(E)** 72

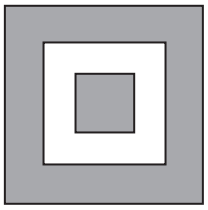
14. A sequence of figures is formed using tiles. Each tile is an equilateral triangle with side length 7 cm. The first figure consists of 1 tile. Each figure after the first is formed by adding 1 tile to the previous figure. The first four figures are as shown:



How many tiles are used to form the figure in the sequence with perimeter 91 cm?

(A) 6 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 23

15. In the diagram, the large square has area 49, the medium square has area 25, and the small square has area 9. The region inside the small square is shaded. The region between the large and medium squares is shaded. What is the total area of the shaded regions?



(A) 33 **(B)** 58 **(C)** 45
(D) 25 **(E)** 13

16. Which of the following expressions is not equivalent to $3x + 6$?

(A) $3(x + 2)$ **(B)** $\dfrac{-9x - 18}{-3}$ **(C)** $\dfrac{1}{3}(3x) + \dfrac{2}{3}(9)$
(D) $\dfrac{1}{3}(9x + 18)$ **(E)** $3x - 2(-3)$

17. Ben participates in a prize draw. He receives one prize that is equally likely to be worth \$5, \$10 or \$20. Jamie participates in a different prize draw. She receives one prize that is equally likely to be worth \$30 or \$40. What is the probability that the total value of their prizes is exactly \$50?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{2}{3}$

18. A positive integer n is a multiple of 7. The square root of n is between 17 and 18. How many possible values of n are there?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

19. Each of the following 15 cards has a letter on one side and a positive integer on the other side.

e	17	57	60	D
43	E	3	7	13
31	88	G	H	21

What is the minimum number of cards that need to be turned over to check if the following statement is true?

> "If a card has a lower case letter on one side, then it has an odd integer on the other side."

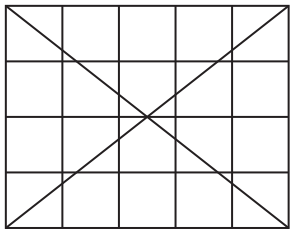
(A) 11 **(B)** 9 **(C)** 7 **(D)** 5 **(E)** 3

20. A large $5 \times 5 \times 5$ cube is formed using 125 small $1 \times 1 \times 1$ cubes. There are three central columns, each passing through the small cube at the very centre of the large cube: one from top to bottom, one from front to back, and one from left to right. All of the small cubes that make up these three columns are removed. What is the surface area of the resulting solid?

(A) 204 **(B)** 206 **(C)** 200 **(D)** 196 **(E)** 192

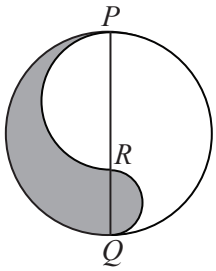
Part C: Each correct answer is worth 8.

21. In the 4×5 grid shown, six of the 1×1 squares are *not* intersected by either diagonal. When the two diagonals of an 8×10 grid are drawn, how many of the 1×1 squares are *not* intersected by either diagonal?



(A) 44 **(B)** 24 **(C)** 52
(D) 48 **(E)** 56

22. In the diagram, PQ is a diameter of a larger circle, point R is on PQ, and smaller semi-circles with diameters PR and QR are drawn. If $PR = 6$ and $QR = 4$, what is the ratio of the area of the shaded region to the area of the unshaded region?



(A) 4 : 9 **(B)** 2 : 3 **(C)** 3 : 5
(D) 2 : 5 **(E)** 1 : 2

23. Ali, Bea, Che, and Deb compete in a checkers tournament. Each player plays each other player exactly once. At the end of each game, either the two players tie or one player wins and the other player loses. A player earns 5 points for a win, 0 points for a loss, and 2 points for a tie. Exactly how many of the following final point distributions are possible?

Player	Points
Ali	15
Bea	7
Che	4
Deb	2

Player	Points
Ali	10
Bea	10
Che	4
Deb	4

Player	Points
Ali	15
Bea	5
Che	5
Deb	2

Player	Points
Ali	12
Bea	10
Che	5
Deb	0

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

24. Lucas chooses one, two or three different numbers from the list $2, 5, 7, 12, 19, 31, 50, 81$ and writes down the sum of these numbers. (If Lucas chooses only one number, this number is the sum.) How many different sums less than or equal to 100 are possible?

(A) 43 **(B)** 39 **(C)** 42 **(D)** 40 **(E)** 41

25. We call the pair (m, n) of positive integers a *happy pair* if the greatest common divisor of m and n is a perfect square. For example, $(20, 24)$ is a happy pair because the greatest common divisor of 20 and 24 is 4. Suppose that k is a positive integer such that $(205\,800, 35k)$ is a happy pair. The number of possible values of k with $k \leq 2940$ is

(A) 36 **(B)** 28 **(C)** 24 **(D)** 30 **(E)** 27

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For students...

Thank you for writing the 2020 Pascal Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
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- Find your school's contest results