



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Hypatia Contest

(Grade 11)

Wednesday, April 15, 2020

(in North America and South America)

Thursday, April 16, 2020

(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 75 minutes

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Do not open this booklet until instructed to do so.

Number of questions: 4

Each question is worth 10 marks

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks



WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x -intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. At a local grocery store, avocados are sold for \$5.00 per bag and mangoes for \$12.50 per box. A bag contains 6 avocados and a box contains 15 mangoes. Only a whole number of bags and a whole number of boxes can be purchased.



- (a) On Friday, a chef purchased 12 bags of avocados and some boxes of mangoes. If the total cost was \$135.00, how many boxes of mangoes were purchased?



- (b) On Saturdays only, there is a 10% discount on the price of a bag of avocados and a 20% discount on the price of a box of mangoes. What is the total cost for 8 bags of avocados and 4 boxes of mangoes on Saturdays?

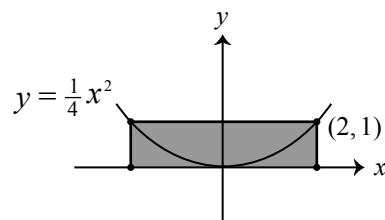


- (c) On Monday, the chef needed 100 avocados and 70 mangoes. The chef purchased just enough bags and boxes. Determine how much the purchase cost her.



- (d) On Tuesday, the chef made special tarts that each required 1 avocado and 2 mangoes. If the chef spent *exactly* \$75.00 on avocados and mangoes, determine the greatest number of tarts that she could have made.

2. The parabola with equation $y = \frac{1}{4}x^2$ has its vertex at the origin and the y -axis as its axis of symmetry. For any point (p, q) on the parabola (not at the origin), we can form a *parabolic rectangle*. This rectangle will have one vertex at (p, q) , a second vertex on the parabola, and the other two vertices on the x -axis. A parabolic rectangle with area 4 is shown.



- (a) A parabolic rectangle has one vertex at $(6, 9)$. What are the coordinates of the other three vertices?



- (b) What is the area of the parabolic rectangle having one vertex at $(-3, 0)$?



- (c) Determine the areas of the two parabolic rectangles that have a side length of 36.

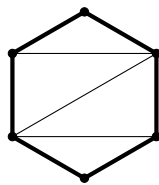


- (d) Determine the area of the parabolic rectangle whose length and width are equal.

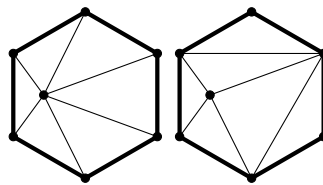
3. A *triangulation* of a regular polygon is a division of its interior into triangular regions. In such a division, each vertex of each triangle is either a vertex of the polygon or an interior point of the polygon. In a triangulation of a regular polygon with $n \geq 3$ vertices and $k \geq 0$ interior points with no three of these $n + k$ points lying on the same line,

- no two line segments connecting pairs of these points cross anywhere except at their endpoints, and
- each interior point is a vertex of at least one of the triangular regions.

Every regular polygon has at least one triangulation. The number of triangles formed by any triangulation of a regular polygon with n vertices and k interior points is constant and is denoted $T(n, k)$. For example, in every possible triangulation of a regular hexagon and one interior point, there are exactly 6 triangles. That is, $T(6, 1) = 6$.



$$T(6, 0) = 4$$



$$T(6, 1) = 6$$



- (a) What is the value of $T(3, 2)$?



- (b) Determine the value of $T(4, 100)$.



- (c) Determine the value of n for which $T(n, n) = 2020$.

4. Let x_0 be a non-negative integer. For each integer $i \geq 0$, define $x_{i+1} = (x_i)^2 + 1$.



- (a) Show that $x_2 - x_0$ is even for all possible values of x_0 .



- (b) Show that $x_{2026} - x_{2020}$ is divisible by 10 for all possible values of x_0 .



- (c) Parsa chooses an integer n with $1 \leq n \leq 100$ at random and sets $x_0 = n$. Determine the probability that $x_{115} - 110$ is divisible by 105.



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For students...

Thank you for writing the 2020 Hypatia Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2020.

Visit our website cemc.uwaterloo.ca to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Obtain information about our 2020/2021 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware
- Learn about our face-to-face workshops and our web resources
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- Find your school's contest results