The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

2020 Gauss Contests

(Grades 7 and 8)

Wednesday, May 13, 2020
(in North America and South America)

Thursday, May 14, 2020
(outside of North America and South America)

Solutions

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Grade 7

1. One pens costs \$2. The cost of 10 pens is $10 \times \$2 = \$20$.

   Answer: (E)

2. Beginning at the origin $(0, 0)$, point P is located right 2 units (in the positive x direction) and up 4 units (in the positive y direction).
   Thus the coordinates of point P are $(2, 4)$.

   Answer: (E)

3. Since 99 is close to 100 and 9 is close to 10, the value of 99×9 is approximately equal to $100 \times 10 = 1000$.
   Of the given answers, this means that the integer that is closest to 99×9 is most likely 1000.
   Multiplying, we get that the value of 99×9 is 891.
   Of the given answers, 99×9 is indeed closest to 1000.

   Answer: (D)

4. The increase in temperature equals the warmer afternoon temperature minus the colder morning temperature.
   The difference between 5 and -3 is $5 - (-3) = 5 + 3 = 8$.
   The temperature increased by 8°C.

   Answer: (A)

5. In April, Alexis averaged $243\,000 \div 30 = 8100$ steps per day.

   Answer: (B)

6. *Solution 1*
   One complete rotation equals 360°.
   Since 90° is $\frac{90^\circ}{360^\circ} = \frac{1}{4}$ of one complete rotation, then $\frac{1}{4}$ of all students chose juice.
   Therefore, the remaining $1 - \frac{1}{4} = \frac{3}{4}$ of all students chose milk.
   Since $\frac{3}{4}$ is $3 \times \frac{1}{4}$, then $3 \times 80 = 240$ students chose milk.

   Solution 2
   As in Solution 1, $\frac{1}{4}$ of all students chose juice.
   Since the 80 students who chose juice represent $\frac{1}{4}$ of the total number of students, then the total number of students is $4 \times 80 = 320$.
   Therefore, $320 - 80 = 240$ students chose milk.

   Answer: (C)

7. Since the third and fourth numbers in the list are consecutive and add to 11, then the third number in the list is 5 and the fourth is 6.
   The fifth number in the list is 7, and so the sixth number in the list is 8.

   Answer: (D)

8. *Solution 1*
   Between 0 and 1.0, the number line is divided by tick marks (at P, Q, R) into 4 equal lengths.
   Thus, the distance between adjacent tick marks is $\frac{1.0-0}{4} = 0.25$.
   Since R is the 3^{rd} tick mark moving right from 0, the value of R is $3 \times 0.25 = 0.75$.
   Since U is the 6^{th} tick mark moving right from 0, the value of U is $6 \times 0.25 = 1.5$.
   The value of R divided by the value of U equals $\frac{0.75}{1.5} = 0.5$.

Solution 2

The number R is located at the 3^{rd} tick mark to the right of 0.
The number U is located at the 6^{th} tick mark to the right of 0.
The tick marks are equally spaced along the number line, and so the value of R must be half the value of U (since 3 is half of 6).
The value of R divided by the value of U equals $\frac{1}{2}$ or 0.5.

Answer: (B)

9. The triangle is isosceles, and so the missing side length is also 12 cm.
The perimeter of the triangle is $14 + 12 + 12 = 38$ cm.
The perimeter of the rectangle is given by $x + 8 + x + 8 = 2x + 16$ cm.
The perimeter of the triangle is equal to the perimeter of the rectangle, and so $2x + 16 = 38$.
Since $22 + 16 = 38$, then $2x = 22$ and so $x = 11$.

Answer: (C)

10. In the table below, we list the divisors of each of the given answers (other than the number itself) and determine their sum.

Given Answers	Divisors	Sum of the Divisors
8	$1, 2, 4$	$1 + 2 + 4 = 7$
10	$1, 2, 5$	$1 + 2 + 5 = 8$
14	$1, 2, 7$	$1 + 2 + 7 = 10$
18	$1, 2, 3, 6, 9$	$1 + 2 + 3 + 6 + 9 = 21$
22	$1, 2, 11$	$1 + 2 + 11 = 14$

The sum of the divisors of each of $8, 10, 14$, and 22 is less than the number itself.
Each of these four answers is not an abundant number. (These numbers are called "deficient".)
The sum of the divisors of 18 is 21, which is greater than 18.
Thus 18 is an abundant number.

Answer: (D)

11. Each of 7 boxes contains exactly 10 cookies, and so the total number of cookies is $7 \times 10 = 70$.
If the cookies are shared equally among 5 people, then each person receives $70 \div 5 = 14$ cookies.

Answer: (A)

12. Abdul is 9 years older than Susie, and Binh is 2 years older than Susie, and so Abdul is $9 - 2 = 7$ years older than Binh.
For example, if Susie is 10 years old, then Abdul is $9 + 10 = 19$, Binh is $2 + 10 = 12$, and Abdul is $19 - 12 = 7$ years older than Binh.

Answer: (E)

13. The y-coordinates of points $P(15, 55)$ and $Q(26, 55)$ are equal.
Therefore, the distance between P and Q is equal to the positive difference between their x-coordinates, or $26 - 15 = 11$.
Similary, the x-coordinates of points $R(26, 35)$ and $Q(26, 55)$ are equal.
Therefore, the distance between R and Q is equal to the positive difference between their y-coordinates, or $55 - 35 = 20$.
Since $PQ = 11$ and $RQ = 20$, the area of rectangle $PQRS$ is $11 \times 20 = 220$.

Answer: (C)

14. Before Jack eats any jelly beans, the box contains $15 + 20 + 16 = 51$ jelly beans.
After Jack eats 2 jelly beans, there are $51 - 2 = 49$ jelly beans remaining in the box.
One of the jelly beans that Jack ate was green, and the other was blue.
Thus after eating the 2 jelly beans, there are still 15 red jelly beans in the box.
If each of the remaining jelly beans is equally likely to be chosen, the probability that Jack chooses a red jelly bean next is $\frac{15}{49}$.

Answer: (C)

15. *Solution 1*

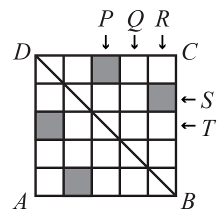
There are 60 minutes in 1 hour, and so there are $60 + 52 = 112$ minutes in 1 hour 52 minutes.
If Emil's race time was 54 minutes, then Olivia's race time was 4 minutes more, or 58 minutes.
In this case, their race times total $54 + 58 = 112$ minutes, as required.
Therefore, it took Olivia 58 minutes to run the race.

Solution 2

As in Solution 1, the total of their race times is 112 minutes.
If Emil's race time was 4 minutes more, then his race time would be equal to Olivia's, and the total of their race times would be $112 + 4 = 116$ minutes.
If their race times are equal and total 116 minutes, then they each finished the race in $116 \div 2 = 58$ minutes.
Therefore, it took Olivia 58 minutes to run the race.

Answer: (C)

16. For BD to be a line of symmetry of $ABCD$, the squares labelled P and S should be shaded.



To see why this is, consider that the reflection in BD of vertex A is vertex C, and so the reflection in BD of side DA is side DC.
Further, the reflection in BD of the first column of square $ABCD$ is the first row of square $ABCD$.
Thus, the reflection in BD of the shaded square in row 3, column 1 is the square in row 1, column 3 (the square labelled P).
More generally, the reflection in BD of the square in row r, column c is the square in row c, column r.
Thus, the reflection in BD of the shaded square in row 5, column 2 is the square in row 2, column 5 (the square labelled S).

Answer: (A)

17. If Rosie deposits \$30 each month for m months, she will save $30m$ dollars.
Rosie has \$120 in her account today, and so after m deposits Rosie's total savings (in dollars) is best represented by the expression $120 + 30m$.

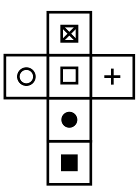
Answer: (E)

18. Isosceles triangles have two equal angles, and so the possibilities for these two triangles are:
1) The two equal angles are each equal to $70°$, or
2) The two equal angles are each not equal to $70°$.
(We note that a triangle can not have three angles measuring $70°$ since the sum of the three angles would be $3 \times 70° = 210°$, which is greater than $180°$.)
If the two equal angles are each equal to $70°$, then the measure of the third angle is $180° - 2 \times 70° = 180° - 140° = 40°$.
If the two equal angles are each not equal to $70°$, then the sum of the measures of the two equal angles is $180° - 70° = 110°$, and so the measure of each of the equal angles is half of $110°$ or $55°$.

We note that in the first triangle, the measure of each of the two remaining angles (70° and 40°) is even, and in the second triangle, the measure of each of the two remaining angles (55° and 55°) is odd.
In the first triangle, the sum of the two equal angles is $S = 70^\circ + 70^\circ = 140^\circ$.
In the second triangle, the sum of the two equal angles is $T = 55^\circ + 55^\circ = 110^\circ$.
The value of $S + T$ is $140^\circ + 110^\circ = 250^\circ$.

ANSWER: (B)

19. We begin by recognizing that there are 6 different symbols, and so each face of the cube contains a different symbol.
From left to right, let us number the views of the cube 1, 2 and 3.
Views 1 and 2 each show a face containing the symbol ⊠ .
What symbol is on the face opposite to the face containing ⊠?
In view 1, □ and ○ are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
In view 2, ■ and + are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
There is only one symbol remaining, and so ● must be the symbol that is on the face opposite ⊠, and vice versa.
A net of the cube is shown below.



ANSWER: (C)

20. On each of her four tosses of the coin, Jane will either move up one dot or she will move right one dot.
Since Jane has two possible moves on each of her four tosses of the coin, she has a total of $2 \times 2 \times 2 \times 2 = 16$ different paths that she may take to arrive at one of P, Q, R, S, or T.
If we denote a move up one dot by U, and a move right one dot by R, these 16 paths are: $UUUU$, $UUUR$, $UURU$, $UURR$, $URUU$, $URUR$, $URRU$, $URRR$, $RUUU$, $RUUR$, $RURU$, $RURR$, $RRUU$, $RRUR$, $RRRU$, $RRRR$.
The probability of tossing a head (and thus moving up one dot) is equal to the probability of tossing a tail (and thus moving right one dot).
That is, it is equally probable that Jane will take any one of these 16 paths.
Therefore, the probability that Jane will finish at dot R is equal to the number of paths that end at dot R divided by the total number of paths, 16.
How many of the 16 paths end at dot R?
Beginning at A, a path ends at R if it has two moves up (two U's), and two moves right (two R's).
There are 6 such paths: $UURR$, $URUR$, $URRU$, $RUUR$, $RURU$, $RRUU$.
(We note that each of the other 10 paths will end at one of the other 4 dots, P, Q, S, T.)
After four tosses of the coin, the probability that Jane will be at dot R is $\frac{6}{16} = \frac{3}{8}$.

ANSWER: (B)

21. Each four-digit number must be greater than 2000, and so the smallest two-digit number that may be repeated is 20 (to give 2020).
Each four-digit number must be less than 10 000, and so the largest two-digit number that may be repeated is 99 (to give 9999).
Each two-digit number between 20 and 99 may be repeated to give a four-digit number between 2000 and 10 000.
In total, there are $99 - 20 + 1 = 80$ numbers that satisfy the given conditions.

ANSWER: (A)

22. Celyna spent \$5.00 on candy A and \$7.00 on candy B, or \$12.00 in total.
The average price of all the candy that she purchased was \$1.50 per 100 grams.
This means that if Celyna bought 100 grams of candy, she would have spent \$1.50.
If she bought 200 grams of candy, she would have spent $2 \times \$1.50 = \$3.00$.
How many grams of candy would Celyna need to buy to spend \$12.00?
Since $8 \times \$1.50 = \$12.00$ (or $\$12.00 \div \$1.50 = 8$), then she would need to buy a total of 800 grams of candy.
Celyna bought 300 grams of candy A, and so she must have purchased $800 - 300 = 500$ grams of candy B. The value of x is 500.

ANSWER: (C)

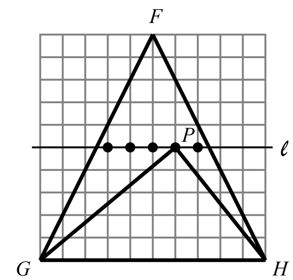
23. If the first positive integer in the list is a and the second is b, then the third integer is $a + b$, the fourth is $b + (a + b)$ or $a + 2b$, and the fifth is $(a + b) + (a + 2b)$ or $2a + 3b$.
Thus, we are asked to find the number of pairs of positive integers a and b, where a is less than b (since the list is increasing), and for which $2a + 3b = 124$.
What is the largest possible value for b?
If $b = 42$, then $3b = 3 \times 42 = 126$ which is too large since $2a + 3b = 124$. (Note that a larger value of b makes $3b$ even larger.)
If $b = 41$, then $3b = 3 \times 41 = 123$.
However in this case, we get that $2a = 124 - 123 = 1$, which is not possible since a is a positive integer.
If $b = 40$, then $3b = 3 \times 40 = 120$ and so $2a = 4$ or $a = 2$.
Thus, the largest possible value for b is 40.
What is the smallest value for b?
If $b = 26$, then $3b = 3 \times 26 = 78$ and so $2a = 124 - 78 = 46$ or $a = 23$.
If $b = 25$, then $3b = 3 \times 25 = 75$.
However in this case, we get that $2a = 124 - 75 = 49$, which is not possible since a is a positive integer.
If $b = 24$, then $3b = 3 \times 24 = 72$ and so $2a = 124 - 72 = 52$ or $a = 26$.
However, if the first integer in the list is 26, then the second integer can not equal 24 since the list is increasing.
Smaller values of b will give larger values of a, and so the smallest possible value of b is 26.
From the values of b attempted thus far, we notice that when b is an odd integer, $3b$ is also odd (since the product of two odd integers is odd), and $124 - 3b$ is odd (since the difference between an even integer and an odd integer is odd).
So when b is odd, $124 - 3b$ is odd, and so $2a$ is odd (since $2a = 124 - 3b$).
However, $2a$ is even for every choice of the integer a and so b cannot be odd.
Conversely, when b is even, $124 - 3b$ is even (as required), and so all even integer values of b from 26 to 40 inclusive will satisfy the requirements.
These values of b are $26, 28, 30, 32, 34, 36, 38, 40$, and so there are 8 such lists of five integers that have 124 as the fifth integer.

Here are the 8 lists: $2, 40, 42, 82, 124$; $5, 38, 43, 81, 124$; $8, 36, 44, 80, 124$; $11, 34, 45, 79, 124$; $14, 32, 46, 78, 124$; $17, 30, 47, 77, 124$; $20, 28, 48, 76, 124$; $23, 26, 49, 75, 124$.

Answer: (E)

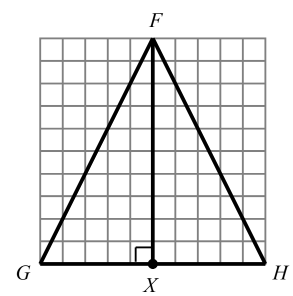
24. We begin by determining the area of $\triangle FGH$.
The base GH has length 10, the perpendicular height of the triangle from GH to F is also 10, and so the area of $\triangle FGH$ is $\frac{1}{2} \times 10 \times 10 = 50$.
We wish to determine which of the 41 points is a possible location for P so that $\triangle FPG$ or $\triangle GPH$ or $\triangle HPF$ has area $\frac{1}{2} \times 50 = 25$.

We begin by considering the possible locations for P so that $\triangle GPH$ has area 25.
The base GH has length 10, and so the perpendicular height from GH to P must be 5 (since $\frac{1}{2} \times 10 \times 5 = 25$).
Since the distance between two parallel lines remains constant, any point P lying on a line that is parallel to GH and that is 5 units from GH will give a $\triangle GPH$ whose area is 25.
The line labelled ℓ is parallel to GH and lies 5 units above GH, and so any point that lies on ℓ is a distance of 5 units from base GH.

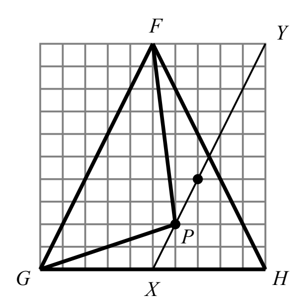


There are 5 points that lie on ℓ (that are at the intersection of gridlines) and that are inside $\triangle FGH$. These 5 points and one of the 5 possibilities for $\triangle GPH$ are shown.

Next, we consider the possible locations for P so that $\triangle FPG$ has area 25.
Consider the point X on GH so that FX is perpendicular to GH.
Since $\triangle FGH$ is isosceles, then FX divides the area of $\triangle FGH$ in half and so $\triangle FXG$ has area 25.
However, if X lies on GH, then X does not lie *inside* $\triangle FGH$.
For this reason, X is not a possible location for P, however it does provide some valuable information and insight.
If we consider the base of $\triangle FXG$ to be FG, then the perpendicular distance from X to FG is equal to the height required from base FG so that $\triangle FXG$ has area 25.



Any point P that lies on a line that is parallel to FG and that is the same distance from FG as X will give a $\triangle FPG$ whose area is also 25.
(This is the same property that we saw previously for $\triangle GPH$, with base GH.)
How do we create a line that passes through X and that is parallel to FG?
Beginning at G, if we move 5 units right and 10 units up we arrive at F.
Begin at X, move 5 units right and 10 units up, and call this point Y.
Can you explain why the line segment XY is parallel to FG?
Any point that lies on XY is a distance from base FG equal to the required height of $\triangle FPG$.
There are 2 points that lie on XY (that are at the intersection of gridlines) and that are inside $\triangle FGH$.



(We note that we may move right 5 and up 10 by moving in 'steps' of right 1 and up 2 to arrive at each of these 2 points.)
These 2 points and one possibility for $\triangle FPG$ are shown.

Finally, we consider the possible locations for P so that $\triangle HPF$ has area 25.
As a result of symmetry, this case is identical to the previous case.

Thus, there are 2 possible locations for P so that $\triangle HPF$ has area 25.
In total, there are $5+2+2=9$ triangles that have an area that is exactly half of the area of $\triangle FGH$.

Answer: (E)

25. Bus A takes 12 minutes to complete one round trip that begins and ends at P.
Since $PX = XS$, it takes Bus A $12 \div 4 = 3$ minutes to travel from P to X, 6 minutes to travel from X to S to X (3 minutes from X to S and 3 minutes from S to X), and 6 minutes to travel from X to P to X.
That is, Bus A first arrives at X at 1:03 and then continues to return to X every 6 minutes.
We write times that Bus A arrives at X in the table below.

Bus A	1:03	1:09	1:15	1:21	1:27	1:33	1:39	1:45	1:51	1:57	2:03

Notice that Bus A arrives at X at 1:03 and exactly one hour later at 2:03.
This makes sense since Bus A returns to X every 6 minutes and 60 minutes (one hour) is divisible by 6.
This tells us that Bus A will continue to arrive at the same number of minutes past each hour, or 2:03, 2:09, 2:15, ..., 3:03, 3:09, ..., 5:03, 5:09, ..., 9:03, 9:09, ..., 9:51, 9:57.

Bus B takes 20 minutes to complete one round trip that begins and ends at Q.
Since $QX = XT$, it takes Bus B $\frac{20}{4} = 5$ minutes to travel from Q to X, 10 minutes to travel from X to T to X (5 minutes from X to T and 5 minutes from T to X), and 10 minutes to travel from X to Q to X.
That is, Bus B first arrives at X at 1:05 and then continues to return to X every 10 minutes.
We write times that Bus B arrives at X in the table below.

Bus B	1:05	1:15	1:25	1:35	1:45	1:55	2:05

Notice that Bus B arrives at X at 1:05 and exactly one hour later at 2:05.
This makes sense since Bus B returns to X every 10 minutes and 60 minutes (one hour) is divisible by 10.
This tells us that Bus B will continue to arrive at the same number of minutes past each hour, or 2:05, 2:15, 2:25, ..., 3:05, 3:15, ..., 5:05, 5:15, ..., 9:05, 9:15, ..., 9:45, 9:55.
From the two tables above, we see that Bus A and Bus B both arrive at X at 15 minutes and 45 minutes past each hour.
Thus between 5:00 p.m. and 10:00 p.m., these two buses will meet $2 \times 5 = 10$ times at X.
These times are: 5:15, 5:45, 6:15, 6:45, 7:15, 7:45, 8:15, 8:45, 9:15, and 9:45.

Bus C takes 28 minutes to complete one round trip that begins and ends at R.
Since $RX = XU$, it takes Bus C $\frac{28}{4} = 7$ minutes to travel from R to X, $2 \times 7 = 14$ minutes to travel from X to U to X, and 14 minutes to travel from X to R to X.
That is, Bus C first arrives at X at 1:07 and then continues to return to X every 14 minutes.
Unlike Bus A and Bus B, Bus C will not arrive at X at consistent times past each hour since 60 is not divisible by 14.
What is the first time after 5:00 p.m. that Bus C arrives at X?
Since 238 is a multiple of 14 ($14 \times 17 = 238$), Bus C will arrive at X 238 minutes after first arriving at X at 1:07 p.m.
Since 238 minutes is 2 minutes less than 4 hours ($4 \times 60 = 240$), Bus C will arrive at X at 5:05 p.m. (This is the first time after 5:00 p.m. that Bus C arrives at X.)

Bus B also arrives at X at 5:05 p.m.
Are there other times after 5:05 p.m. (and before 10:00 p.m.) that Bus B and Bus C arrive at X at the same time?
Bus B arrives at X every 10 minutes and Bus C arrives at X every 14 minutes.
Since the lowest common multiple of 10 and 14 is 70, then Bus B and Bus C will each arrive at X every 70 minutes after 5:05 p.m., or at 6:15 p.m., 7:25 p.m., 8:35 p.m., and at 9:45 p.m.
Next we determine if there are times when Bus A and Bus C arrive at X at the same time.
Bus C arrives at X every 14 minutes after 5:05 p.m., or 5:19 p.m., 5:33 p.m., and so on.
Bus A also arrives at X at 5:33 p.m.
Are there other times after 5:33 p.m. (and before 10:00 p.m.) that Bus A and Bus C arrive at X at the same time?
Bus A arrives at X every 6 minutes and Bus C arrives at X every 14 minutes.
Since the lowest common multiple of 6 and 14 is 42, then Bus A and Bus C will each arrive at X every 42 minutes after 5:33 p.m., or at 6:15 p.m., 6:57 p.m., 7:39 p.m., 8:21 p.m., 9:03 p.m., and at 9:45 p.m.
The times when each pair of buses meet at X at the same time between 5:00 p.m. and 10:00 p.m. are listed below.
Bus A and Bus B: 15 and 45 minutes past each hour
Bus B and Bus C: 5:05 p.m., 6:15 p.m., 7:25 p.m., 8:35 p.m., 9:45 p.m.
Bus A and Bus C: 5:33 p.m., 6:15 p.m., 6:57 p.m., 7:39 p.m., 8:21 p.m., 9:03 p.m., 9:45 p.m.
Finally, we determine the number of different times that two or more buses arrive at X at the same time.
Bus A and Bus B arrive at X at 10 different times.
Bus B and Bus C arrive at X at 5 different times; however 2 of these times (6:15 p.m. and 9:45 p.m.) have already been counted, so there are 3 new times.
Bus A and Bus C arrive at X at 7 different times; however 2 of these times (6:15 p.m. and 9:45 p.m.) have already been counted, so there are 5 new times.
The number of times that two or more buses arrive at X between 5:00 p.m. and 10:00 p.m. is $10 + 3 + 5 = 18$.

Answer: (A)

Grade 8

1. Including 1 with the four given numbers and ordering the list, we get $-0.2, 0.03, 0.76, 1, 1.5$. Thus, there are 3 numbers in the list which are less than 1 (these are $-0.2, 0.03$ and 0.76).

   Answer: (D)

2. If the total cost of 4 one-litre cartons of milk is \$4.88, then the cost of 1 one-litre carton of milk is $\$4.88 \div 4 = \$1.22$.

   Answer: (E)

3. Of the answers given, $\frac{12}{2} = 6$ is the only fraction which is equal to a whole number.

   Answer: (E)

4. Since $x + y = 0$, then x and y must be opposites.
   Since $x = 4$, then $y = -4$, and we may check that $4 + (-4) = 0$.

   Answer: (E)

5. The length of the base is the distance from the origin to the point $(4, 0)$ or 4.
   The height is the distance from the origin to the point $(0, 6)$ or 6.
   Thus, the area of the triangle is $\frac{1}{2} \times 4 \times 6 = 12$.
   (We note that the base and height are perpendicular.)

   Answer: (A)

6. The whole numbers between 2 and 20 whose square root is a whole number are: $4, 9, 16$ (since $\sqrt{4} = 2, \sqrt{9} = 3, \sqrt{16} = 4$).
   We may note that perfect squares are equal to the square of an integer.
   That is, the smallest five positive perfect squares are $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16$ and $5^2 = 25$.
   Of these five, there are three that are between 2 and 20.

   Answer: (D)

7. *Solution 1*
   For each of the 4 different notebooks that Yvonn may choose, there are 5 different pens that may be chosen. Thus, there are $4 \times 5 = 20$ possible combinations of notebooks and pens that he could bring to class.

   Solution 2
   We begin by naming the 4 notebooks A, B, C, D, and numbering the 5 pens $1, 2, 3, 4, 5$.
   If Yvonn chooses notebook A and pen number 1, then we may call this combination $A1$.
   If Yvonn chooses notebook A, he may instead choose one of the pens numbered $2, 3, 4$, or 5.
   If Yvonn chooses notebook A, he has a total of 5 possible combinations: $A1, A2, A3, A4, A5$.
   Similarly, Yvonn has 5 pen choices for each choice of notebook.
   These remaining possible combinations are: $B1, B2, B3, B4, B5, C1, C2, C3, C4, C5, D1, D2, D3, D4, D5$.
   Thus, there are 20 possible combinations of notebooks and pens that Yvonn could bring to class.

   Answer: (C)

8. We begin by recognizing that a right angle is marked in the pie chart.
   Since 90° is $\frac{1}{4}$ of a full (360°) rotation, then $\frac{1}{4}$ of the students chose apples as their favourite fruit. If $\frac{1}{4}$ of the students chose apples, then the remaining $1 - \frac{1}{4} = \frac{3}{4}$ of the students chose bananas. If 168 students represent $\frac{3}{4}$ of all students, then $\frac{1}{4}$ of all students is $168 \div 3 = 56$ students. Thus, 56 students chose apples as their favourite fruit.

   Answer: (B)

9. There are 8 letters in the bag and 2 of these letters are B's.
If Elina randomly chooses one of the 8 letters, then the probability that she chooses a B is $\frac{2}{8} = \frac{1}{4}$.

Answer: (A)

10. Balil's result, b, is 5 more than Vita's number.
Cali's result, c, is 5 less than Vita's number.
Thus the difference between Balil's result and Cali's result, $b - c$, is 10.
For example, if Vita chooses 8, then $b = 8 + 5 = 13$ and $c = 8 - 5 = 3$ and $b - c = 13 - 3 = 10$.

Answer: (E)

11. The bus stops at the library at 1:00 p.m., 1:20 p.m., and 1:40 p.m..
The bus stops at the library at 2:00 p.m., 2:20 p.m., and 2:40 p.m..
Similarly, the bus stops at the library at x:00 p.m., x:20 p.m., and x:40 p.m. for $x = 3, 4, 5$.
Finally, the bus stops at 6:00 p.m..
In total, the bus stops at the library $3 \times 5 + 1 = 16$ times.

Answer: (A)

12. *Solution 1*
Beginning with the units (ones) column, the units digit of the sum $R + R$ is 2.
Thus, $R = 1$ or $R = 6$ are the only possibilities ($1 + 1 = 2$ and $6 + 6 = 12$ have units digit 2).
If $R = 1$, then there is no 'carry' from the units column to the tens column.
In this case, the units digit of the sum $Q + Q$ is 1.
This is not possible since $Q + Q = 2Q$ which is an even number for all possible digits Q.
Thus, $R = 6$, which gives $R + R = 12$, and so the 'carry' from the units column to the tens column is 1.
In the tens column the units digit of the sum $Q + Q + 1$ is 1, and so the units digit of $Q + Q$ is 0. Thus, $Q = 0$ or $Q = 5$ are the only possibilities ($0 + 0$ and $5 + 5$ have units digit 0).
If $Q = 0$, then there is no 'carry' from the tens column to the hundreds column.
This is not possible since the sum of the hundreds column is 10 and P is a digit and so cannot be 10.
Thus, $Q = 5$, which gives $Q + Q + 1 = 11$, and so the 'carry' from the tens column to the hundreds column is 1.
The sum of the hundreds column is 10, and so $P + 1 = 10$ or $P = 9$.
The value of $P + Q + R$ is $9 + 5 + 6 = 20$.

Solution 2
QR is a 2-digit number and thus less than 100.
The sum of PQR and QR is greater than 1000, and so PQR must be greater than 900 which means that $P = 9$.
Since the sum of the hundreds column is 10, the 'carry' from the tens column must be 1.
Considering the tens and units (ones) columns together, we see that the result of $QR + QR$ has units (ones) digit 2, tens digit 1, and hundreds digit 1 (since there is a 'carry' of 1 to the hundreds column). That is, $QR + QR = 112$ and so $QR = 56$.
The value of $P + Q + R$ is $9 + 5 + 6 = 20$.

Answer: (E)

13. *Solution 1*
There are 60 minutes in 1 hour, and so there are $60 + 52 = 112$ minutes in 1 hour 52 minutes.
If Emil's race time was 54 minutes, then Olivia's race time was 4 minutes more, or 58 minutes.
In this case, their race times total $54 + 58 = 112$ minutes, as required.
Therefore, it took Olivia 58 minutes to run the race.

Solution 2

As in Solution 1, the total of their race times is 112 minutes.
If Emil's race time was 4 minutes more, then his race time would be equal to Olivia's, and the total of their race times would be $112 + 4 = 116$ minutes.
If their race times are equal and total 116 minutes, then they each finished the race in $116 \div 2 = 58$ minutes. Therefore, it took Olivia 58 minutes to run the race.

ANSWER: (C)

14. In $\triangle ABC$, $\angle ABC = 90^\circ$ and so by the Pythagorean Theorem, $BC^2 = 34^2 - 16^2 = 1156 - 256$ or $BC^2 = 900$ and so $BC = \sqrt{900} = 30$ m.
The perimeter of $ABCD$ is $16 + 30 + 16 + 30 = 92$ m.

ANSWER: (D)

15. If Francesca first chooses -4, then there is no integer that she may choose second to give a sum of 3 (the largest integer in the list is 6, and $-4 + 6 = 2$).
If she first chooses -3, she may then choose 6 to give a sum of 3.
If she first chooses -2, she may then choose 5 to give a sum of 3.
If she first chooses -1, she may then choose 4 second to give a sum of 3.
If she first chooses 0, she may then choose 3 to give a sum of 3.
If she first chooses 1, she may then choose 2 to give a sum of 3.
If Francesca first chooses 2 (or any integer larger than 2), then there is no integer that she may choose second to give a sum of 3 (the second integer must be larger than the first, and so the sum will be larger than 5).
There are 5 such pairs of integers that Francesca can choose so that the sum is 3.

ANSWER: (B)

16. Since $\triangle QRS$ is an isosceles right-angled triangle with $QR = SR$, then $\angle RQS = \angle RSQ = 45^\circ$.
Opposite angles are equal in measure, and so $\angle SUV = \angle PUQ = y^\circ$ and $\angle SVU = \angle RVT = y^\circ$.
In $\triangle SVU$, $\angle VSU + \angle SUV + \angle SVU = 180^\circ$ or $45^\circ + y^\circ + y^\circ = 180^\circ$ or $2y = 135$ and so $y = 67.5$.

ANSWER: (C)

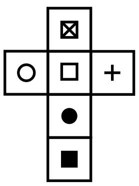
17. In a list of five numbers ordered from smallest to largest, the median is equal to the third (middle) number in the list.
Since x and y must be integers, the values of x and y must belong to exactly one of the following three possibilities:

- each of x and y is less than or equal to 11, or
- each of x and y is greater than or equal to 13, or
- at least one of x or y is equal to 12.

If each of x and y is less than or equal to 11, then the median number in the list is 11.
(In this case, the ordered list could be $x, y, 11, 12, 13$ or $y, x, 11, 12, 13$.)
If each of x and y is greater than or equal to 13, then the median number in the list is 13.
(In this case, the ordered list could be $11, 12, 13, x, y$ or $11, 12, 13, y, x$.)
If at least one of x or y is equal to 12, then the list includes $11, 12, 12, 13$ and one other number.
When the list is ordered, the two 12s will either be the 2nd and 3rd numbers in the list, or the 3rd and 4th numbers in the list depending on whether the unknown number is less than or equal to 12 or greater than or equal to 12.
In either case, the median is 12.
Thus, there are three different possible medians for Mark's five point totals.

ANSWER: (C)

18. We begin by recognizing that there are 6 different symbols, and so each face of the cube contains a different symbol.
From left to right, let us number the views of the cube 1, 2 and 3.
Views 1 and 2 each show a face containing the symbol ⊠ .
What symbol is on the face opposite to the face containing ⊠?
In view 1, □ and ○ are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
In view 2, ■ and + are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
There is only one symbol remaining, and so ● must be the symbol that is on the face opposite ⊠, and vice versa.
A net of the cube is shown below.



Answer: (C)

19. X is 20% of 50, and so $X = 0.20 \times 50 = 10$.
20% of 100 is 20, and so 20% of 200 is 40. Thus $Y = 200$.
40 is Z% of 50, and so $Z = \frac{40}{50} \times 100 = 80$.
(We may check that 80% of 50 is indeed $0.80 \times 50 = 40$.)
Therefore, $X + Y + Z = 10 + 200 + 80$ or $X + Y + Z = 290$.

Answer: (D)

20. We begin by expressing $\frac{20}{19}$ in a form that is similar to the right side of the given equation.
Converting $\frac{20}{19}$ to a mixed fraction we get, $\frac{20}{19} = 1\frac{1}{19} = 1 + \frac{1}{19}$.
Since $\frac{20}{19} = 1 + \dfrac{1}{1 + \frac{a}{b}}$ and $\frac{20}{19} = 1 + \frac{1}{19}$, then $1 + \dfrac{1}{1 + \frac{a}{b}} = 1 + \frac{1}{19}$ and so $\dfrac{1}{1 + \frac{a}{b}} = \frac{1}{19}$.
The numerators of $\dfrac{1}{1 + \frac{a}{b}}$ and $\frac{1}{19}$ are each equal to 1, and since these fractions are equal to one another, their denominators must also be equal.
That is, $1 + \frac{a}{b} = 19$ and so $\frac{a}{b} = 18$.
Since a and b are positive integers, then the fractions $\frac{a}{b}$ which are equal to 18 are $\frac{18}{1}$, $\frac{36}{2}$, $\frac{54}{3}$, and so on.
Thus, the least possible value of $a + b$ is $18 + 1 = 19$.

Answer: (B)

21. Originally, the ratio of green balls to yellow balls in the bag was $3 : 7$.
This means that for every 3 green balls in the bag, there were 7 yellow balls.
Equivalently, if there were $3n$ green balls, then there were $7n$ yellow balls where n is a positive integer.
After 9 balls of each colour are removed, the number of green balls in the bag is $3n - 9$ and the number of yellow balls is $7n - 9$.
At this point, the ratio of green balls to yellow balls is $1 : 3$, and so 3 times the number of green balls is equal to the number of yellow balls.
Multiplying the number of green balls by 3, we get $3 \times 3n - 3 \times 9$ or $9n - 27$ green balls.
Solving the equation $9n - 27 = 7n - 9$, we get $9n - 7n = 27 - 9$ or $2n = 18$, and so $n = 9$.

Originally, there were $3n$ green balls and $7n$ yellow balls, for a total of $3n + 7n = 10n$ which is $10 \times 9 = 90$ balls.
Note: If there were 90 balls, then 27 were green and 63 were yellow (since $27 : 63 = 3 : 7$ and $27 + 63 = 90$). After 9 balls of each colour are removed, the ratio of green balls to yellow balls becomes $18 : 54 = 1 : 3$, as required.

ANSWER: (B)

22. A number is divisible by 6 if it is divisible by both 2 and 3.
To be divisible by 2, the three-digit number that is formed must be even and so the ones digit must be 0 or 2.
To be divisible by 3, the sum of the digits of the number must be a multiple of 3.
Consider the possible tens and hundreds digits when the ones digit is 0.
In this case, the sum of the tens and hundreds digits must be a multiple of 3 (since the ones digit does not add anything to the sum of the digits).
We determine the possible sums of the tens and hundreds digits in the table below.
The sums which are a multiple of 3 are circled.

The Tens Digit

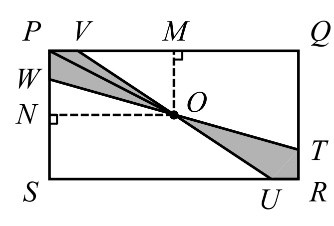
The Hundreds Digit

100s \ 10s	5	6	7	8
1	(6)	7	8	(9)
2	7	8	(9)	10
3	8	(9)	10	11
4	(9)	10	11	(12)

When the ones digit is 0, the possible three-digit numbers are: $150, 180, 270, 360, 450$, and 480.
Consider the possible tens and hundreds digits when the ones digit is 2.
In this case, the sum of the tens and hundreds digits must be 2 less than a multiple of 3 (since the ones digit adds 2 to the sum of the digits).
When the ones digit is 2, the possible three-digit numbers are: $162, 252, 282, 372$, and 462.
The number of three-digit numbers that can be formed that are divisible by 6 is 11.

ANSWER: (A)

23. We begin by joining the centre of the rectangle, O, to vertex P.
We also draw OM perpendicular to side PQ and ON perpendicular to side PS.
Since O is the centre of the rectangle, then M is the midpoint of side PQ and so $PM = \frac{1}{2} \times 4 = 2$.
Similarly, N is the midpoint of PS and so $PN = \frac{1}{2} \times 2 = 1$.



$\triangle PVO$ has base $PV = a$ and height $OM = 1$, and so has area $\frac{1}{2} \times a \times 1 = \frac{1}{2}a$.
$\triangle PWO$ has base $PW = a$ and height $ON = 2$, and so has area $\frac{1}{2} \times a \times 2 = a$.
Thus, quadrilateral $PWOV$ has area equal to the sum of the areas of these two triangles, or $\frac{1}{2}a + a = \frac{3}{2}a$.

Similarly, we can show that quadrilateral $RTOU$ also has area $\frac{3}{2}a$ and so the total area of the shaded region is $2 \times \frac{3}{2}a = 3a$.
The area of rectangle $PQRS$ is $4 \times 2 = 8$ and since the area of the shaded region is $\frac{1}{8}$ the area of $PQRS$, then $3a = \frac{1}{8} \times 8$ or $3a = 1$ and so $a = \frac{1}{3}$.

ANSWER: (D)

24. Bus A takes 12 minutes to complete one round trip that begins and ends at P.
Since $PX = XS$, it takes Bus A $12 \div 4 = 3$ minutes to travel from P to X, 6 minutes to travel from X to S to X (3 minutes from X to S and 3 minutes from S to X), and 6 minutes to travel from X to P to X.
That is, Bus A first arrives at X at 1:03 and then continues to return to X every 6 minutes.
We write times that Bus A arrives at X in the table below.

Bus A	1:03	1:09	1:15	1:21	1:27	1:33	1:39	1:45	1:51	1:57	2:03

Notice that Bus A arrives at X at 1:03 and exactly one hour later at 2:03.
This makes sense since Bus A returns to X every 6 minutes and 60 minutes (one hour) is divisible by 6.
This tells us that Bus A will continue to arrive at the same number of minutes past each hour, or 2:03, 2:09, 2:15, ..., 3:03, 3:09, ..., 5:03, 5:09, ..., 9:03, 9:09, ..., 9:51, 9:57.

Bus B takes 20 minutes to complete one round trip that begins and ends at Q.
Since $QX = XT$, it takes Bus B $\frac{20}{4} = 5$ minutes to travel from Q to X, 10 minutes to travel from X to T to X (5 minutes from X to T and 5 minutes from T to X), and 10 minutes to travel from X to Q to X.
That is, Bus B first arrives at X at 1:05 and then continues to return to X every 10 minutes.
We write times that Bus B arrives at X in the table below.

Bus B	1:05	1:15	1:25	1:35	1:45	1:55	2:05

Notice that Bus B arrives at X at 1:05 and exactly one hour later at 2:05.
This makes sense since Bus B returns to X every 10 minutes and 60 minutes (one hour) is divisible by 10.
This tells us that Bus B will continue to arrive at the same number of minutes past each hour, or 2:05, 2:15, 2:25, ..., 3:05, 3:15, ..., 5:05, 5:15, ..., 9:05, 9:15, ..., 9:45, 9:55.
From the two tables above, we see that Bus A and Bus B both arrive at X at 15 minutes and 45 minutes past each hour.
Thus between 5:00 p.m. and 10:00 p.m., these two buses will meet $2 \times 5 = 10$ times at X.
These times are: 5:15, 5:45, 6:15, 6:45, 7:15, 7:45, 8:15, 8:45, 9:15, and 9:45.

Bus C takes 28 minutes to complete one round trip that begins and ends at R.
Since $RX = XU$, it takes Bus C $\frac{28}{4} = 7$ minutes to travel from R to X, $2 \times 7 = 14$ minutes to travel from X to U to X, and 14 minutes to travel from X to R to X.
That is, Bus C first arrives at X at 1:07 and then continues to return to X every 14 minutes.
Unlike Bus A and Bus B, Bus C will not arrive at X at consistent times past each hour since 60 is not divisible by 14.
What is the first time after 5:00 p.m. that Bus C arrives at X?
Since 238 is a multiple of 14 ($14 \times 17 = 238$), Bus C will arrive at X 238 minutes after first arriving at X at 1:07 p.m.
Since 238 minutes is 2 minutes less than 4 hours ($4 \times 60 = 240$), Bus C will arrive at X at 5:05 p.m. (This is the first time after 5:00 p.m. that Bus C arrives at X.)

Bus B also arrives at X at 5:05 p.m.
Are there other times after 5:05 p.m. (and before 10:00 p.m.) that Bus B and Bus C arrive at X at the same time?
Bus B arrives at X every 10 minutes and Bus C arrives at X every 14 minutes.
Since the lowest common multiple of 10 and 14 is 70, then Bus B and Bus C will each arrive at X every 70 minutes after 5:05 p.m., or at 6:15 p.m., 7:25 p.m., 8:35 p.m., and at 9:45 p.m.
Next we determine if there are times when Bus A and Bus C arrive at X at the same time.
Bus C arrives at X every 14 minutes after 5:05 p.m., or 5:19 p.m., 5:33 p.m., and so on.
Bus A also arrives at X at 5:33 p.m.
Are there other times after 5:33 p.m. (and before 10:00 p.m.) that Bus A and Bus C arrive at X at the same time?
Bus A arrives at X every 6 minutes and Bus C arrives at X every 14 minutes.
Since the lowest common multiple of 6 and 14 is 42, then Bus A and Bus C will each arrive at X every 42 minutes after 5:33 p.m., or at 6:15 p.m., 6:57 p.m., 7:39 p.m., 8:21 p.m., 9:03 p.m., and at 9:45 p.m.
The times when each pair of buses meet at X at the same time between 5:00 p.m. and 10:00 p.m. are listed below.
Bus A and Bus B: 15 and 45 minutes past each hour
Bus B and Bus C: 5:05 p.m., 6:15 p.m., 7:25 p.m., 8:35 p.m., 9:45 p.m.
Bus A and Bus C: 5:33 p.m., 6:15 p.m., 6:57 p.m., 7:39 p.m., 8:21 p.m., 9:03 p.m., 9:45 p.m.
Finally, we determine the number of different times that two or more buses arrive at X at the same time.
Bus A and Bus B arrive at X at 10 different times.
Bus B and Bus C arrive at X at 5 different times; however 2 of these times (6:15 p.m. and 9:45 p.m.) have already been counted, so there are 3 new times.
Bus A and Bus C arrive at X at 7 different times; however 2 of these times (6:15 p.m. and 9:45 p.m.) have already been counted, so there are 5 new times.
The number of times that two or more buses arrive at X between 5:00 p.m. and 10:00 p.m. is $10 + 3 + 5 = 18$.

Answer: (A)

25. The property of an integer being either even or odd is called its *parity*.
If two integers are both even or they are both odd, then we say that the two integers have the *same parity*.
If one integer is even and a second integer is odd, then we say that the two integers have *different parity*.
The result of adding two integers that have the same parity is an even integer.
The result of adding two integers that have different parity is an odd integer.
The parity of each term of an FT sequence (after the second term) is determined by the parity of the first two terms in the sequence.
For example, if each of the first two terms of an FT sequence is odd, then the third term is even (since odd plus odd is even), the fourth term is odd (since odd plus even is odd), the fifth term is odd (since even plus odd is odd), and so on.
There are 4 possibilities for the parities of the first two terms of an FT sequence.
The sequence could begin odd, odd, or even, even, or odd, even, or even, odd.
In the table below, we write the parity of the first few terms of the FT sequences that begin in each of the 4 possible ways.

Term Number	1	2	3	4	5	6	7	8	9	10
Parity #1	odd	odd	even	odd	odd	even	odd	odd	even	odd
Parity #2	even	even	even	even	even	even	even	even	even	even
Parity #3	odd	even	odd	odd	even	odd	odd	even	odd	odd
Parity #4	even	odd	odd	even	odd	odd	even	odd	odd	even

The FT sequence beginning odd, odd (Parity #1) continues to repeat odd, odd, even.
Since the parity of each term is dependent on the parity of the two terms preceding it, this odd, odd, even pattern will continue throughout the entire sequence.
That is, in each successive group of three terms beginning at the first term, one out of three terms will be even and two out of three terms will be odd.
The odd, odd, even pattern ends at term numbers that are multiples of 3 (the even-valued terms are terms 3, 6, 9, 12, and so on).
Since 2019 is a multiple of 3 ($2019 = 3 \times 673$), $\frac{1}{3}$ of the first 2019 terms will be even-valued and $\frac{2}{3}$ will be odd-valued, and so there are twice as many odd-valued terms as there are even-valued terms in the first 2019 terms.
The 2020^{th} term is odd (since the pattern begins with an odd-valued term), and so there are more than twice as many odd-valued terms as there are even-valued terms in every FT sequence that begins odd, odd.
This is exactly the required condition for the FT sequences that we are interested in.
How many FT sequences begin with two odd-valued terms, each of which is a positive integer less than $2m$?
There are $2m - 1$ positive integers less than $2m$ (these are $1, 2, 3, 4, \ldots, 2m - 1$).
Since m is a positive integer, $2m$ is always an even integer and so $2m - 1$ is always odd.
Thus, the list of integers from 1 to $2m - 1$ begins and ends with an odd integer, and so the list contains m odd integers and $m - 1$ even integers.
The first term in the sequence is odd-valued and so there are m choices for it.
Similarly, the second term in the sequence is also odd-valued and so there are also m choices for it.
Thus, there are a total of $m \times m$ or m^2 FT sequences that begin with two odd-valued terms.

Do any of the other 3 types of FT seqences satisfy the required condition that there are more than twice as many odd-valued terms as there are even-valued terms?
Clearly the FT sequence beginning even, even (Parity #2) does not satisfy the required condition since every term in the sequence is even-valued.
The FT sequence beginning odd, even (Parity #3) continues to repeat odd, even, odd.
That is, in each successive group of three terms beginning at the first term, one out of three terms will be even and two out of three terms will be odd.
As we saw previously, 2019 is a multiple of 3 and so $\frac{1}{3}$ of the first 2019 terms are even-valued and $\frac{2}{3}$ are odd-valued.
Thus there are twice as many odd-valued terms as there are even-valued terms in the first 2019 terms.
The 2020^{th} term is odd (since the pattern begins with an odd-valued term), and so there are more than twice as many odd-valued terms as there are even-valued terms in every FT sequence that begins odd, even.
How many FT sequences begin with an odd-valued first term and an even-valued second term, each being a positive integer less than $2m$?
As we showed previously, the list of integers from 1 to $2m - 1$ begins and ends with an odd-valued integer, and so the list contains m odd-valued integers and $m - 1$ even-valued integers.
The first term in the sequence is odd-valued and so there are m choices for it.

The second term in the sequence is even-valued and so there are $m - 1$ choices for it.
Thus, there are a total of $m \times (m - 1)$ FT sequences that begin with an odd-valued term followed by an even-valued term.

Finally, we consider the FT sequences that begin with an even-valued term followed by an odd-valued term (Parity #4).
Again, there are exactly twice as many odd-valued terms as there are even-valued terms in the first 2019 terms (since the pattern repeats even, odd, odd).
However in this case, the 2020^{th} term is even and so there are fewer than twice as many odd-valued terms as there are even-valued terms.

Thus, there are $m^2 + m \times (m - 1)$ FT sequences that satisfy the required conditions.
Since there are 2415 such FT sequences, we may solve $m^2 + m \times (m - 1) = 2415$ by trial and error.
Evaluating $m^2 + m \times (m - 1)$ when $m = 30$, we get $30^2 + 30 \times 29 = 1770$, and so m is greater than 30.
When $m = 33$, we get $33^2 + 33 \times 32 = 2145$.
When $m = 34$, we get $34^2 + 34 \times 33 = 2278$.
When $m = 35$, we get $35^2 + 35 \times 34 = 2415$, as required.

ANSWER: (D)