The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Galois Contest

(Grade 10)

Wednesday, April 15, 2020
(in North America and South America)

Thursday, April 16, 2020
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by 
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. The letters A and B are used to create a pattern consisting of a number of rows. The pattern starts with a single A. The rows alternate between A's and B's, and the number of letters in each row is twice the number of letters in the previous row. The first 4 rows of the pattern are shown.

Row 1	A
Row 2	BB
Row 3	$AAAA$
Row 4	$BBBBBBBB$

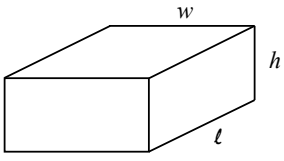

(a) If the pattern consists of 6 rows, how many letters are in the 6^{th} row of the pattern?


(b) If the pattern consists of 6 rows, what is the total number of letters in the pattern?


(c) If the total number of letters in the pattern is 63, determine the number of A's in the pattern and the number of B's in the pattern.


(d) If the total number of letters in the pattern is 4095, determine the difference between the number of A's and the number of B's in the pattern.

2. For a rectangular prism with length ℓ, width w, and height h as shown, the surface area is given by the formula $A = 2\ell w + 2\ell h + 2wh$ and the volume is given by the formula $V = \ell wh$.




(a) What is the surface area of a rectangular prism with length 2 cm, width 5 cm, and height 9 cm?


(b) A rectangular prism with height 10 cm has a square base. The volume of the prism is 160 cm^3. What is the side length of the square base?


(c) A rectangular prism has a square base with area 36 cm^2. The surface area of the prism is 240 cm^2. Determine the volume of the prism.


(d) A rectangular prism has length k cm, width $2k$ cm, and height $3k$ cm, where $k > 0$. The volume of the prism is x cm^3. The surface area of the prism is x cm^2. Determine the value of k.

3. Jodi multiplied the numbers 2 and 5 to get a product of 10. She added 4 to each of her original numbers to get 6 and 9. She multiplied these new numbers to get a product of 54. Jodi noticed that each of the digits in the new product, 54, was 4 more than the corresponding digits in the first product, 10.

$$\begin{array}{c} 2 \times 5 = 10 \\ {\scriptstyle +4}\downarrow \ {\scriptstyle +4}\downarrow \ {\scriptstyle +4}\downarrow\downarrow{\scriptstyle +4} \\ 6 \times 9 = 54 \end{array}$$

The pair $(2, 5)$ is an example of a *RadPair*.
In general, a pair of positive integers (a, b) with $a \leq b \leq 9$ and for which the product ab is a two-digit integer is called a *RadPair* if there exists a positive integer d such that

- the product $(a + d)(b + d)$ is a two-digit integer, and
- the ones (units) digit of the product $(a + d)(b + d)$ equals d plus the ones digit of the product ab, and
- the tens digit of the product $(a + d)(b + d)$ equals d plus the tens digit of the product ab.

(a) Show that $(2, 8)$ is a RadPair.

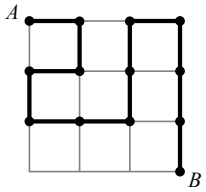
(b) Show that $(3, 6)$ is not a RadPair.

(c) For which positive integers x with $x \leq 6$ is $(x, 6)$ a RadPair?

(d) Determine, with justification, the number of RadPairs (a, b) with $a \leq b$.

4. In an $n \times n$ grid of unit squares, each point at which two grid lines meet is called a *vertex*, and so there are $(n + 1)^2$ *vertices*. The top left corner vertex is labeled A and the bottom right corner vertex is labeled B. A *path* from A to B is a sequence of unit edges that

- each connect two adjacent vertices, and
- when connected, form a sequence of vertices that begins at A, ends at B, and
- passes through each vertex at most once.



The length of such a path is the number of unit edges in the path. For example, in a 3×3 grid, a path of length 12 between A and B is shown.

(a) In a 2×2 grid, determine the number of paths of any length from A to B.

(b) Explain why there cannot be a path from A to B of odd length in a 10×10 grid.

(c) In a 4×4 grid, determine the number of paths of length 10 from A to B.

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For students...

Thank you for writing the 2020 Galois Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2020.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2020/2021 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
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- Find your school's contest results