The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Wednesday, April 15, 2020
(in North America and South America)

Thursday, April 16, 2020
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by 
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

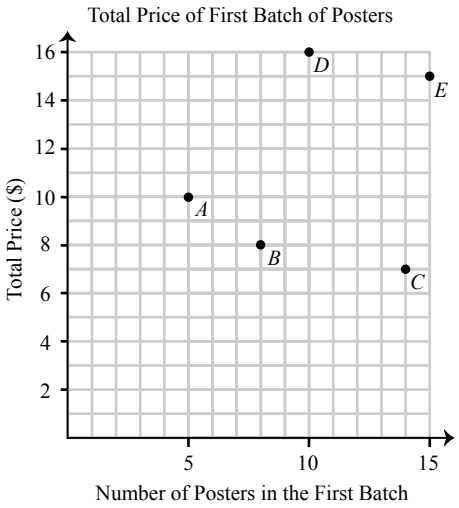
Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. Annie (A), Bogdan (B), Cao (C), Daniel (D), and Emily (E) are running for student government positions. Their campaign posters are printed by different printing companies. The graph to the right plots each student's total price of their first batch of posters versus the number of posters in their first batch.



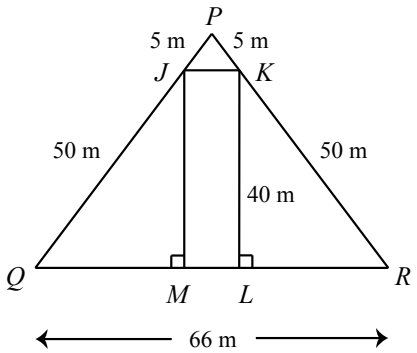
 (a) Daniel pays \$16.00 for 10 posters which is a price of \$1.60 per poster. What does Cao pay per poster?

 (b) Which two students are paying the same price per poster?

 (c) In Daniel's second batch, he needs to print 40 posters. He can pay \$60.00 to print these 40 posters at his local library. To spend less money on the second batch, should Daniel print these 40 posters at the library, or should he continue to pay the same price per poster using the company that printed his first batch?

 (d) Annie's printing company charged \$10.00 for her first 5 posters. Her company will charge her a lower price per poster for additional posters. Emily's printing company charges a fixed price per poster for any number of posters. By the end of the campaign, Annie and Emily each have printed 25 posters and have spent the same amount of money. What price is Annie charged per additional poster?

2. In the diagram, rectangle $JKLM$ is drawn with its vertices on the sides of $\triangle PQR$ so that $PJ = PK = 5$ m, $JQ = KR = 50$ m, $KL = 40$ m, and $QR = 66$ m, as shown.



(a) What is the length of LR?

(b) What is the length of ML?

(c) Determine the height of $\triangle PJK$ drawn from P to JK.

(d) Determine the fraction of the area of $\triangle PQR$ that is covered by rectangle $JKLM$.

3. In a *Dlin sequence*, the first term is a positive integer and each term after the first is calculated by adding 1 to the previous term in the sequence, then doubling the result. For example, the first seven terms of the Dlin sequence with first term 4 are:

$$4, 10, 22, 46, 94, 190, 382$$

(a) The 5^{th} term in a Dlin sequence is 142. What are the 4^{th} and 6^{th} terms in the sequence?

(b) Determine all possible first terms which give a Dlin sequence that includes 1406.

(c) Which possible first terms from 10 to 19 inclusive produce a Dlin sequence in which all terms after the first have the same ones (units) digit?

(d) Determine the number of positive integers between 1 and 2020, inclusive, that can be the third term in a Dlin sequence.

4. An $m \times n$ grid has m rows and n columns. Each cell in the grid is coloured either red (R) or blue (B). For example, a 1×2 grid can be coloured in 4 different ways, as shown below.

R	B	B	R	B	B	R	R

(a) How many different ways can a 5×1 grid be coloured so that exactly 3 cells are red and 2 cells are blue?

(b) Carrie writes down all possible colourings of a 1×13 grid. Looking at her first 1×13 grid, Carrie counts the number of cells coloured red and the number of cells coloured blue, and begins a list by writing down the maximum of these two numbers. Carrie continues this process and writes down this maximum for each of the 1×13 grids that she has coloured. What is the *smallest* number in Carrie's list?

(c) Determine the smallest value of n so that no matter how a $3 \times n$ grid is coloured, it must have at least two columns that are coloured in an identical way.

(d) Consider the following statement:

> In a 5×41 grid, we can always find 3 rows and 3 columns such that the 9 cells located at the intersections of these 3 rows and 3 columns are all the same colour.

Determine whether the statement above is true or false and justify your answer.

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For students...

Thank you for writing the 2020 Fryer Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2020.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2020/2021 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
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- Find your school's contest results