The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Tuesday, February 25, 2020
(in North America and South America)

Wednesday, February 26, 2020
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

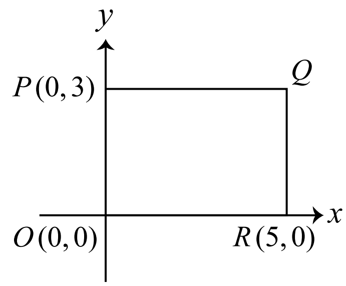
Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The points $O(0,0)$, $P(0,3)$, Q, and $R(5,0)$ form a rectangle, as shown. The coordinates of Q are



(A) $(5,5)$ **(B)** $(5,3)$ **(C)** $(3,3)$
(D) $(2.5,1.5)$ **(E)** $(0,5)$

2. The value of $3 \times 2020 + 2 \times 2020 - 4 \times 2020$ is

(A) 6060 **(B)** 4040 **(C)** 8080 **(D)** 0 **(E)** 2020

3. For every real number x, the expression $(x+1)^2 - x^2$ is equal to

(A) $2x+1$ **(B)** $2x-1$ **(C)** $(2x+1)^2$ **(D)** -1 **(E)** $x+1$

4. Ewan writes out a sequence where he counts by 11s starting at 3. The resulting sequence is $3, 14, 25, 36, \ldots$. A number that will appear in Ewan's sequence is

(A) 113 **(B)** 111 **(C)** 112 **(D)** 110 **(E)** 114

5. The value of $\sqrt{\dfrac{\sqrt{81}+\sqrt{81}}{2}}$ is

(A) 3 **(B)** 18 **(C)** 27 **(D)** 81 **(E)** 162

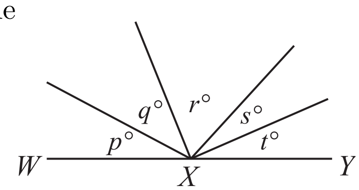
6. Anna thinks of an integer.

- It is *not* a multiple of three.
- It is *not* a perfect square.
- The sum of its digits is a prime number.

The integer that Anna is thinking of could be

(A) 12 **(B)** 14 **(C)** 16 **(D)** 21 **(E)** 26

7. In the diagram, WXY is a straight angle. What is the average (mean) of p, q, r, s, and t?



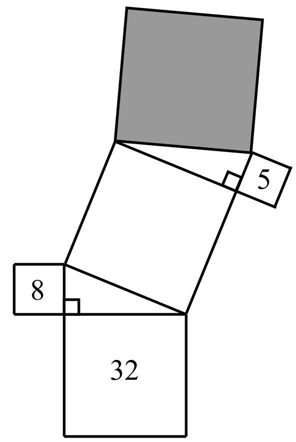
(A) 30 **(B)** 36 **(C)** 60
(D) 72 **(E)** 45

8. If $2^n = 8^{20}$, what is the value of n?

(A) 10 **(B)** 60 **(C)** 40 **(D)** 16 **(E)** 17

9. The figure consists of five squares and two right-angled triangles. The areas of three of the squares are 5, 8 and 32, as shown. What is the area of the shaded square?

(A) 35 **(B)** 45 **(C)** 29
(D) 19 **(E)** 75



10. Positive integers s and t have the property that $s(s - t) = 29$. What is the value of $s + t$?

(A) 1 **(B)** 28 **(C)** 57 **(D)** 30 **(E)** 29

Part B: Each correct answer is worth 6.

11. In the 5×5 grid shown, 15 cells contain X's and 10 cells are empty. Any X may be moved to any empty cell. What is the smallest number of X's that must be moved so that each row and each column contains exactly three X's?

X	X	X	X	
X	X	X		X
X	X			
X	X		X	
		X	X	

(A) 1 **(B)** 2 **(C)** 3
(D) 4 **(E)** 5

12. Harriet ran a 1000 m course in 380 seconds. She ran the first 720 m of the course at a constant speed of 3 m/s. She ran the remaining part of the course at a constant speed of v m/s. What is the value of v?

(A) 2 **(B)** 1.5 **(C)** 3 **(D)** 1 **(E)** 4.5

13. In the list $2, x, y, 5$, the sum of any two adjacent numbers is constant. The value of $x - y$ is

(A) 1 **(B)** -3 **(C)** 3 **(D)** -1 **(E)** 0

14. In Rad's garden there are exactly 30 red roses, exactly 19 yellow roses, and no other roses. How many of the yellow roses does Rad need to remove so that $\frac{2}{7}$ of the roses in the garden are yellow?

(A) 5 **(B)** 6 **(C)** 4 **(D)** 8 **(E)** 7

15. Suppose that $N = 3x + 4y + 5z$, where x equals 1 or -1, and y equals 1 or -1, and z equals 1 or -1. How many of the following statements are true?

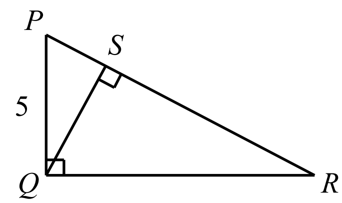
- N can equal 0.
- N is always odd.
- N cannot equal 4.
- N is always even.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

16. Suppose that x and y are real numbers with $-4 \leq x \leq -2$ and $2 \leq y \leq 4$. The greatest possible value of $\dfrac{x+y}{x}$ is

(A) 1 **(B)** -1 **(C)** $-\frac{1}{2}$ **(D)** 0 **(E)** $\frac{1}{2}$

17. In the diagram, $\triangle PQR$ is right-angled at Q and point S is on PR so that QS is perpendicular to PR. If the area of $\triangle PQR$ is 30 and $PQ = 5$, the length of QS is



(A) $\frac{60}{13}$ **(B)** 5 **(C)** $\frac{30}{13}$
(D) 4 **(E)** 3

18. Four teams play in a tournament in which each team plays exactly one game against each of the other three teams. At the end of each game, either the two teams tie or one team wins and the other team loses. A team is awarded 3 points for a win, 0 points for a loss, and 1 point for a tie. If S is the sum of the points of the four teams after the tournament is complete, which of the following values can S *not* equal?

(A) 13 **(B)** 17 **(C)** 11 **(D)** 16 **(E)** 15

19. When $(3 + 2x + x^2)(1 + mx + m^2x^2)$ is expanded and fully simplified, the coefficient of x^2 is equal to 1. What is the sum of all possible values of m?

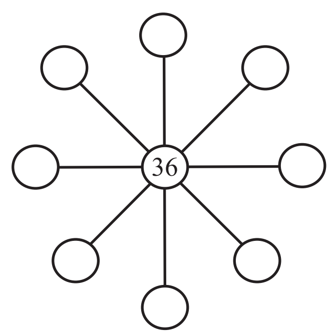
(A) $-\frac{4}{3}$ **(B)** $-\frac{2}{3}$ **(C)** 0 **(D)** $\frac{2}{3}$ **(E)** $\frac{4}{3}$

20. A cube has six faces. Each face has some dots on it. The numbers of dots on the six faces are 2, 3, 4, 5, 6, and 7. Harry removes one of the dots at random, with each dot equally likely to be removed. When the cube is rolled, each face is equally likely to be the top face. What is the probability that the top face has an odd number of dots on it?

(A) $\frac{4}{7}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{13}{27}$ **(D)** $\frac{11}{21}$ **(E)** $\frac{3}{7}$

Part C: Each correct answer is worth 8.

21. In the diagram, the central circle contains the number 36. Positive integers are to be written in the eight empty circles, one number in each circle, so that the product of the three integers along any straight line is 2592. If the nine integers in the circles must be all different, what is the largest possible sum of these nine integers?



(A) 160 **(B)** 176 **(C)** 178
(D) 195 **(E)** 216

22. Suppose that x and y are real numbers that satisfy the two equations:

$$x^2 + 3xy + y^2 = 909$$
$$3x^2 + xy + 3y^2 = 1287$$

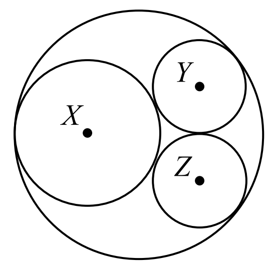
What is a possible value for $x + y$?

(A) 27 **(B)** 39 **(C)** 29 **(D)** 92 **(E)** 41

23. There are real numbers a and b for which the function f has the properties that $f(x) = ax + b$ for all real numbers x, and $f(bx + a) = x$ for all real numbers x. What is the value of $a + b$?

(A) 2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** -2

24. In the diagram, the circle with centre X is tangent to the largest circle and passes through the centre of the largest circle. The circles with centres Y and Z are each tangent to the other three circles, as shown. The circle with centre X has radius 1. The circles with centres Y and Z each have radius r. The value of r is closest to



(A) 0.93 **(B)** 0.91 **(C)** 0.95

(D) 0.87 **(E)** 0.89

25. Three real numbers x, y, z are chosen randomly, and independently of each other, between 0 and 1, inclusive. What is the probability that each of $x - y$ and $x - z$ is greater than $-\frac{1}{2}$ and less than $\frac{1}{2}$?

(A) $\frac{3}{4}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

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For students...

Thank you for writing the 2020 Fermat Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Hypatia Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results