



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Pascal Contest

(Grade 9)

Tuesday, February 26, 2019
(in North America and South America)

Wednesday, February 27, 2019
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

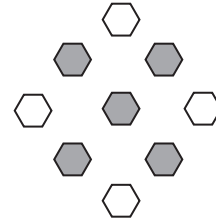
Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

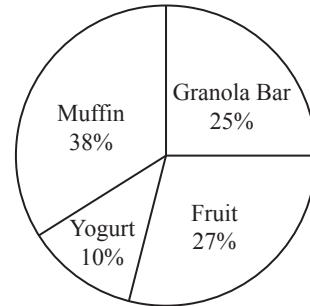
1. The expression $2 \times 3 + 2 \times 3$ equals
 (A) 10 (B) 20 (C) 36 (D) 12 (E) 16

2. The perimeter of a square is 28. What is the side length of this square?
 (A) 9 (B) 6 (C) 8 (D) 4 (E) 7

3. In the diagram, some of the hexagons are shaded. What fraction of all of the hexagons are shaded?
 (A) $\frac{1}{2}$ (B) $\frac{5}{9}$ (C) $\frac{4}{5}$
 (D) $\frac{1}{3}$ (E) $\frac{5}{6}$

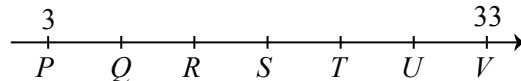


4. Yesterday, each student at Pascal C.I. was given a snack. Each student received either a muffin, yogurt, fruit, or a granola bar. No student received more than one of these snacks. The percentages of the students who received each snack are shown in the circle graph. What percentage of students *did not* receive a muffin?
 (A) 27% (B) 38% (C) 52%
 (D) 62% (E) 78%



5. What is the smallest integer that can be placed in the box so that $\frac{1}{2} < \frac{\square}{9}$?
 (A) 7 (B) 3 (C) 4 (D) 5 (E) 6
6. If $4x + 14 = 8x - 48$, what is the value of $2x$?
 (A) 17 (B) 31 (C) 35 (D) 24 (E) 36

7. In the diagram, point P is on the number line at 3 and V is at 33. The number line between 3 and 33 is divided into six equal parts by the points Q, R, S, T, U .

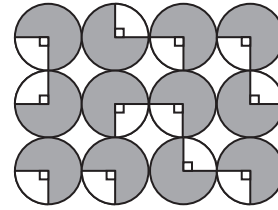


What is the sum of the lengths of PS and TV ?

- (A) 25 (B) 23 (C) 24 (D) 21 (E) 27
8. The median of the numbers in the list $19^{20}, \frac{20}{19}, 20^{19}, 2019, 20 \times 19$ is
 (A) 19^{20} (B) $\frac{20}{19}$ (C) 20^{19} (D) 2019 (E) 20×19

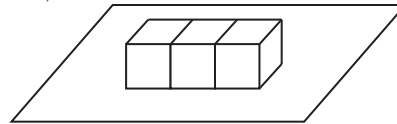
9. In the diagram, each partially shaded circle has a radius of 1 cm and has a right angle marked at its centre. In cm^2 , what is the total shaded area?

(A) $4\pi^2$ (B) $9\pi^2$ (C) 4π
 (D) 9π (E) 3π



10. Three $1 \times 1 \times 1$ cubes are joined face to face in a single row and placed on a table, as shown. The cubes have a total of 11 exposed 1×1 faces. If sixty $1 \times 1 \times 1$ cubes are joined face to face in a single row and placed on a table, how many 1×1 faces are exposed?

(A) 125 (B) 220 (C) 182
 (D) 239 (E) 200



Part B: Each correct answer is worth 6.

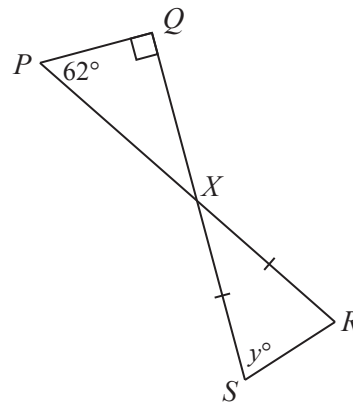
11. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. In the magic square shown, the value of x is

(A) 3.8 (B) 3.6 (C) 3.1
 (D) 2.9 (E) 2.2

2.3		
3.6	3	2.4
	x	

12. In the diagram, PR and QS meet at X . Also, $\triangle PQX$ is right-angled at Q with $\angle QPX = 62^\circ$ and $\triangle RXS$ is isosceles with $RX = SX$ and $\angle XSR = y^\circ$. The value of y is

(A) 54 (B) 71 (C) 76
 (D) 59 (E) 60



13. The list p, q, r, s consists of four consecutive integers listed in increasing order. If $p + s = 109$, the value of $q + r$ is

(A) 108 (B) 109 (C) 110 (D) 117 (E) 111

14. Many of the students in M. Gamache's class brought a skateboard or a bicycle to school yesterday. The ratio of the number of skateboards to the number of bicycles was $7 : 4$. There were 12 more skateboards than bicycles. How many skateboards and bicycles were there in total?

(A) 44 (B) 33 (C) 11 (D) 22 (E) 55

15. Sophie has written three tests. Her marks were 73%, 82% and 85%. She still has two tests to write. All tests are equally weighted. Her goal is an average of 80% or higher. With which of the following pairs of marks on the remaining tests will Sophie *not* reach her goal?

(A) 79% and 82% (B) 70% and 91% (C) 76% and 86%
 (D) 73% and 83% (E) 61% and 99%

16. If x is a number less than -2 , which of the following expressions has the least value?

(A) x (B) $x + 2$ (C) $\frac{1}{2}x$ (D) $x - 2$ (E) $2x$

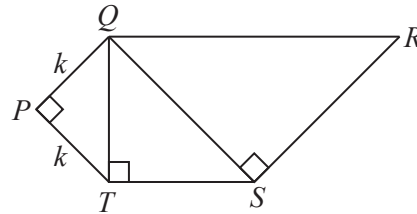
17. Hagrid has 100 animals. Among these animals,

- each is either striped or spotted but not both,
- each has either wings or horns but not both,
- there are 28 striped animals with wings,
- there are 62 spotted animals, and
- there are 36 animals with horns.

How many of Hagrid's spotted animals have horns?

(A) 8 (B) 10 (C) 2 (D) 38 (E) 26

18. In the diagram, each of $\triangle QPT$, $\triangle QTS$ and $\triangle QSR$ is an isosceles, right-angled triangle, with $\angle QPT = \angle QTS = \angle QSR = 90^\circ$. The combined area of the three triangles is 56. If $QP = PT = k$, what is the value of k ?

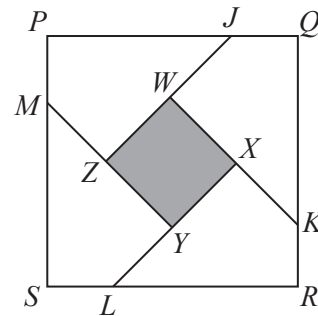


(A) $\sqrt{2}$ (B) 1 (C) 4
 (D) 2 (E) $2\sqrt{2}$

19. There are six identical red balls and three identical green balls in a pail. Four of these balls are selected at random and then these four balls are arranged in a line in some order. How many different-looking arrangements are possible?

(A) 15 (B) 16 (C) 10 (D) 11 (E) 12

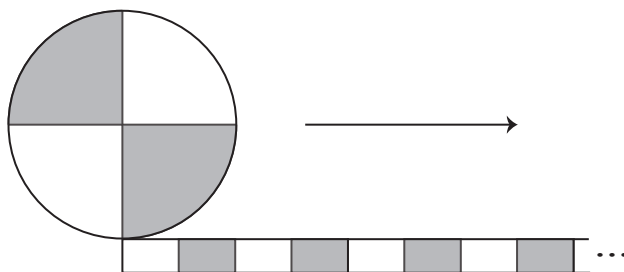
20. In the diagram, square $PQRS$ has side length 40. Points J , K , L , and M are on the sides of $PQRS$, as shown, so that $JQ = KR = LS = MP = 10$. Line segments JZ , KW , LX , and MY are drawn parallel to the diagonals of the square so that W is on JZ , X is on KW , Y is on LX , and Z is on MY . What is the area of quadrilateral $WXYZ$?



(A) 280 (B) 200 (C) 320
 (D) 240 (E) 160

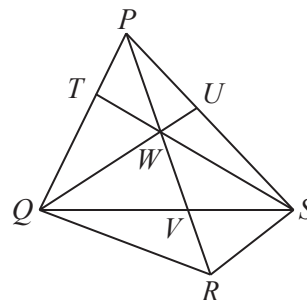
Part C: Each correct answer is worth 8.

21. What is the units (ones) digit of the integer equal to $5^{2019} - 3^{2019}$?
 (A) 0 (B) 2 (C) 4 (D) 6 (E) 8
22. The integer 2019 can be formed by placing two consecutive two-digit positive integers, 19 and 20, in decreasing order. What is the sum of all four-digit positive integers greater than 2019 that can be formed in this way?
 (A) 476 681 (B) 476 861 (C) 478 661 (D) 468 671 (E) 468 761
23. A path of length 14 m consists of 7 unshaded stripes, each of length 1 m, alternating with 7 shaded stripes, each of length 1 m. A circular wheel of radius 2 m is divided into four quarters which are alternately shaded and unshaded. The wheel rolls at a constant speed along the path from the starting position shown.



- The wheel makes exactly 1 complete revolution. The percentage of time during which a shaded section of the wheel is touching a shaded part of the path is closest to
 (A) 20% (B) 18% (C) 16% (D) 24% (E) 22%
24. If p , q , r , and s are digits, how many of the 14-digit positive integers of the form $88\,663\,311\,pqr\,s48$ are divisible by 792?
 (A) 48 (B) 56 (C) 40 (D) 60 (E) 50

25. In the diagram, PR and QS intersect at V . Also, W is on PV , U is on PS and T is on PQ with QU and ST passing through W . For some real number x ,
- the area of $\triangle PUW$ equals $4x + 4$,
 - the area of $\triangle SUW$ equals $2x + 20$,
 - the area of $\triangle SVW$ equals $5x + 20$,
 - the area of $\triangle SVR$ equals $5x + 11$,
 - the area of $\triangle QVR$ equals $8x + 32$, and
 - the area of $\triangle QVW$ equals $8x + 50$.



- The area of $\triangle PTW$ is closest to
 (A) 35 (B) 34 (C) 33
 (D) 32 (E) 31



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For students...

Thank you for writing the 2019 Pascal Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website cemc.uwaterloo.ca to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results