The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Wednesday, April 10, 2019
(in North America and South America)

Thursday, April 11, 2019
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by 
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

Useful Fact:
It may be helpful to know that the sum of the n integers from 1 to n equals $\frac{1}{2}n(n+1)$; that is, $1 + 2 + 3 + \cdots + (n-1) + n = \frac{1}{2}n(n+1)$.

1.  (a) A rectangle with dimenions 7 by 8 is shown in Figure A. What is the perimeter of this figure?

 (b) A 3 by 1 rectangle is removed from one corner of a 7 by 8 rectangle, as shown in Figure B. What is the perimeter of this figure?

 (c) A 4 by 2 rectangle is removed from one corner of a $k+4$ by $k+2$ rectangle, as shown in Figure C. Suppose that the perimeter of Figure C is 56. Determine the value of the integer k.

 (d) Four 4 by 7 rectangles are removed from the corners of a square having side length $8n+1$, as shown in Figure D. Determine the largest integer n for which the perimeter of Figure D is less than 1000.

Figure A

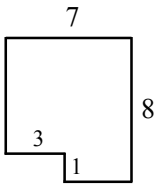

Figure B

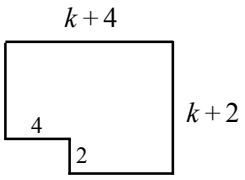

Figure C

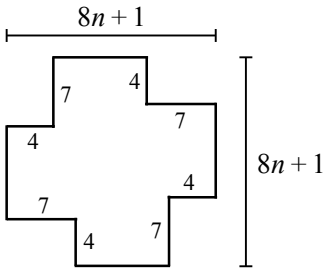

Figure D

2. Rope is fed into a machine at a constant rate of 2 metres per second. The machine can be set to cut off one piece of rope every t seconds for various values of t. For example, if the machine is set to make one cut every 5 seconds, then 12 pieces of rope are cut off in 1 minute.


(a) If the machine is set to make one cut every 8 seconds, how many pieces of rope are cut off in 10 minutes?


(b) If the machine is set to make one cut every 3 seconds, what is the length of each piece of rope that is cut off?


(c) If each piece of rope that is cut off is 30 m long, determine the number of cuts per minute that the machine is set to make.


(d) If the machine is set to make 16 cuts per minute, determine the length of each piece of rope that is cut off.

3. Tania lists the positive integers, in order, leaving out the integers that are multiples of 5. Her resulting list is

$$1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, \ldots.$$


(a) How many integers has Tania listed just before she leaves out the 6th multiple of 5?


(b) If the kth integer in Tania's list is 2019, determine the value of k.


(c) Determine the sum of the first 200 integers in Tania's list.

4. A *Shonk sequence* is a sequence of positive integers in which

- each term after the first is greater than the previous term, and
- the product of all terms is a perfect square.

For example: $2, 6, 27$ is a Shonk sequence since $6 > 2$ and $27 > 6$ and $2 \times 6 \times 27 = 324 = 18^2$.


(a) If $12, x, 24$ is a Shonk sequence, what is the value of x?


(b) If $28, y, z, 65$ is a Shonk sequence, what are the values of y and z?


(c) Determine the length of the longest Shonk sequence, each of whose terms is an integer between 1 and 12, inclusive. This means that your solution should include an example of a sequence of this longest length, as well as justification as to why no longer sequence is possible.


(d) A sequence of four terms a, b, c, d is called a *super-duper-Shonkolistic sequence* (SDSS) exactly when each of a, b, c, d and a, b, c and b, c, d is a Shonk sequence. Determine the number of pairs (m, n) such that $m, 1176, n, 48\,400$ is an SDSS.

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For students...

Thank you for writing the 2019 Fryer Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2019.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2019/2020 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
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