



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Tuesday, February 26, 2019
(in North America and South America)

Wednesday, February 27, 2019
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 60 minutes

©2018 University of Waterloo

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D, and E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

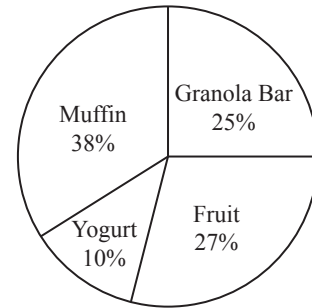
The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

- The expression $2 \times 0 + 1 - 9$ equals
 (A) -8 (B) -6 (C) -7 (D) -11 (E) 0
- Kai will celebrate his 25th birthday in March 2020. In what year was Kai born?
 (A) 1975 (B) 1990 (C) 1995 (D) 2000 (E) 1955

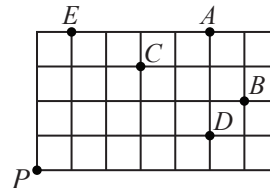
- Yesterday, each student at Cayley S.S. was given a snack. Each student received either a muffin, yogurt, fruit, or a granola bar. No student received more than one of these snacks. The percentages of the students who received each snack are shown in the circle graph. What percentage of students *did not* receive a muffin?



- (A) 27% (B) 38% (C) 52%
 (D) 62% (E) 78%

- The expression $(2 \times \frac{1}{3}) \times (3 \times \frac{1}{2})$ equals
 (A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) 6
- If $10d + 8 = 528$, then $2d$ is equal to
 (A) 104 (B) 76 (C) 96 (D) 41 (E) 520
- The line with equation $y = x + 4$ is translated down 6 units. The y -intercept of the resulting line is
 (A) 6 (B) 4 (C) 10 (D) -6 (E) -2
- The three numbers 2, x , and 10 have an average of x . What is the value of x ?
 (A) 5 (B) 4 (C) 7 (D) 8 (E) 6

- Alain travels on the 4×7 grid shown from point P to one of the points A , B , C , D , or E . Alain can travel only right or up, and only along gridlines. To which point should Alain travel in order to travel the shortest distance?

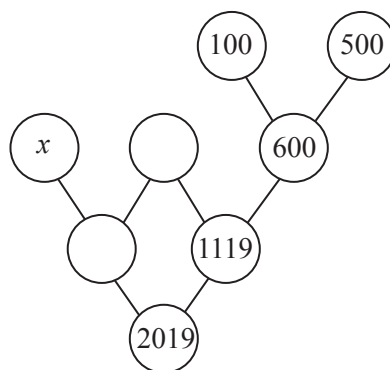


- (A) A (B) B (C) C
 (D) D (E) E

- If $(pq)(qr)(rp) = 16$, then a possible value for pqr is
 (A) 0 (B) 2 (C) 4 (D) 8 (E) 16
- Matilda and Ellie divide a white wall in their bedroom in half, each taking half of the wall. Matilda paints half of her section red. Ellie paints one third of her section red. The fraction of the entire wall that is painted red is
 (A) $\frac{5}{12}$ (B) $\frac{2}{5}$ (C) $\frac{2}{3}$ (D) $\frac{1}{6}$ (E) $\frac{1}{2}$

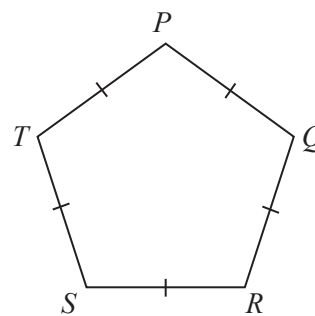
Part B: Each correct answer is worth 6.

11. In the diagram, numbers are to be placed in the circles so that each circle that is connected to two circles above it will contain the sum of the numbers contained in the two circles above it. What is the value of x ?



- (A) 481 (B) 381 (C) 281
(D) 581 (E) 681

12. In a regular pentagon, the measure of each interior angle is 108° . If $PQRST$ is a regular pentagon, then the measure of $\angle PRS$ is



- (A) 72° (B) 54° (C) 60°
(D) 45° (E) 80°

13. In the addition problem shown, m , n , p , and q represent positive digits. When the problem is completed correctly, the value of $m + n + p + q$ is

$$\begin{array}{r} n63 \\ 7p2 \\ + 58q \\ \hline m042 \end{array}$$

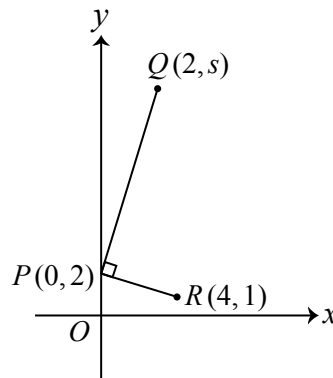
- (A) 23 (B) 24 (C) 21
(D) 22 (E) 20

14. The letters A, B, C, D, and E are to be placed in the grid so that each of these letters appears exactly once in each row and exactly once in each column. Which letter will go in the square marked with *?

A				E
		C	A	
E		B	C	
	*			
B			D	

- (A) A (B) B (C) C
(D) D (E) E

15. In the diagram, the line segments PQ and PR are perpendicular. The value of s is

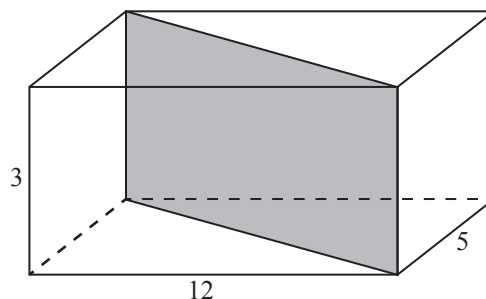


- (A) 6 (B) 9 (C) 10
(D) 12 (E) 9.5

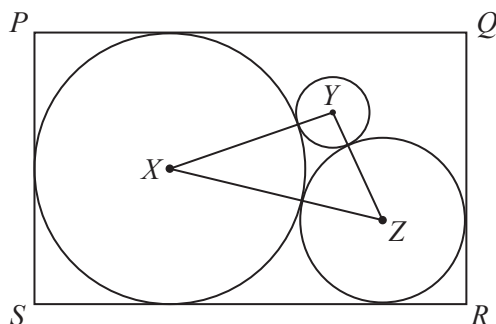
16. Kaukab is standing in a cafeteria line. In the line, the number of people that are ahead of her is equal to two times the number of people that are behind her. There are n people in the line. A possible value of n is

- (A) 23 (B) 20 (C) 24 (D) 21 (E) 25

17. A solid wooden rectangular prism measures $3 \times 5 \times 12$. The prism is cut in half by a vertical cut through four vertices, as shown. This cut creates two congruent triangular-based prisms. When these prisms are pulled apart, what is the surface area of one of these triangular-based prisms?



- (A) 135 (B) 111 (C) 114
 (D) 150 (E) 90
18. Carl and André are running a race. Carl runs at a constant speed of x m/s. André runs at a constant speed of y m/s. Carl starts running, and then André starts running 20 s later. After André has been running for 10 s, he catches up to Carl. The ratio $y : x$ is equivalent to
- (A) 20 : 1 (B) 2 : 1 (C) 1 : 3 (D) 3 : 1 (E) 1 : 2
19. If x and y are positive integers with $xy = 6$, the sum of all of the possible values of $\frac{2^{x+y}}{2^{x-y}}$ is
- (A) 4180 (B) 4160 (C) 4164 (D) 4176 (E) 4128
20. In the diagram, each of the circles with centres X , Y and Z is tangent to the two other circles. Also, the circle with centre X touches three sides of rectangle $PQRS$ and the circle with centre Z touches two sides of rectangle $PQRS$, as shown.



If $XY = 30$, $YZ = 20$ and $XZ = 40$, the area of rectangle $PQRS$ is closest to

- (A) 3900 (B) 4100 (C) 4050 (D) 4000 (E) 3950

Part C: Each correct answer is worth 8.

21. In the multiplication shown, each of P , Q , R , S , and T is a digit. The value of $P + Q + R + S + T$ is

- (A) 14 (B) 20 (C) 16
 (D) 17 (E) 13

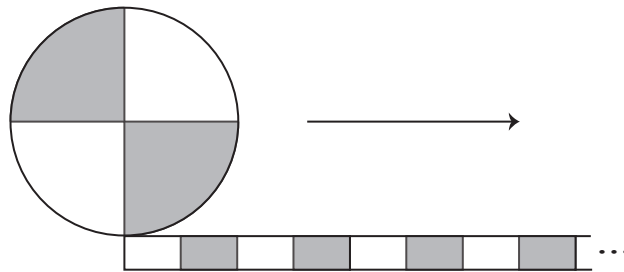
$$\begin{array}{r} P Q R S T 4 \\ \times \quad \quad \quad 4 \\ \hline 4 P Q R S T \end{array}$$

22. Seven friends are riding the bus to school:
- Cha and Bai are on 2 different buses.
 - Bai, Abu and Don are on 3 different buses.
 - Don, Gia and Fan are on 3 different buses.
 - Abu, Eva and Bai are on 3 different buses.
 - Gia and Eva are on 2 different buses.
 - Fan, Cha and Gia are on 3 different buses.
 - Cha and Eva are on 2 different buses.

What is the least possible number of buses on which the friends could be riding?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

23. A path of length 38 m consists of 19 unshaded stripes, each of length 1 m, alternating with 19 shaded stripes, each of length 1 m. A circular wheel of radius 2 m is divided into four quarters which are alternately shaded and unshaded. The wheel rolls at a constant speed along the path from the starting position shown.



The wheel makes exactly 3 complete revolutions. The percentage of time during which a shaded section of the wheel is touching a shaded part of the path is closest to

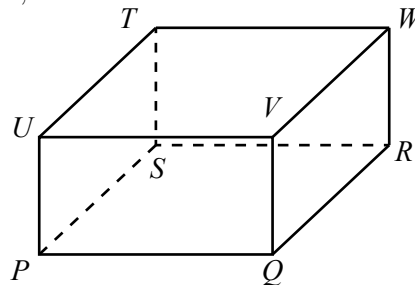
- (A) 20% (B) 18% (C) 24% (D) 22% (E) 26%

24. Roberta chooses an integer r from the set $\{2, 3, 4, 5, 6, 7, 8, 9\}$, an integer s from the set $\{22, 33, 44, 55, 66, 77, 88, 99\}$, and an integer t from the set $\{202, 303, 404, 505, 606, 707, 808, 909\}$. How many possible values are there for the product rst ?

- (A) 85 (B) 81 (C) 90 (D) 84 (E) 80

25. For how many positive integers x does there exist a rectangular prism $PQRSTU VW$, labelled as shown, with $PR = 1867$, $PV = 2019$, and $PT = x$?

- (A) 1980 (B) 1982 (C) 1984
 (D) 1983 (E) 1981





The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

For students...

Thank you for writing the 2019 Cayley Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website cemc.uwaterloo.ca to find

- More information about the Galois Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results