The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

2018 Gauss Contests

(Grades 7 and 8)

Wednesday, May 16, 2018
(in North America and South America)

Thursday, May 17, 2018
(outside of North America and South America)

Solutions

Centre for Education in Mathematics and Computing Faculty and Staff

Gauss Contest Committee

Grade 7

1. Since $21 - 13 = 8$, the number that should be subtracted from 21 to give 8 is 13.

   Answer: (B)

2. Reading from the pie chart, 20% of 100 students chose banana.
   Since 20% of 100 is 20, then 20 students chose banana.

   Answer: (D)

3. There are 30 minutes between 8:30 a.m. and 9:00 a.m.
   There are 5 minutes between 9:00 a.m. and 9:05 a.m.
   Therefore, the length of the class is $30 + 5 = 35$ minutes.

   Answer: (C)

4. The side length of a square having an area of 144 cm^2 is $\sqrt{144}$ cm or 12 cm.

   Answer: (D)

5. The cost of nine \$1 items and five \$2 items is $(9 \times \$1) + (5 \times \$2)$, which is $\$9 + \$10$ or \$19.
   The correct answer is (C).
   (We may check that each of the remaining four answers gives a cost that is less than \$18.)

   Answer: (C)

6. Converting each of the improper fractions to a mixed fraction, we get $\frac{5}{2} = 2\frac{1}{2}$, $\frac{11}{4} = 2\frac{3}{4}$, $\frac{11}{5} = 2\frac{1}{5}$, $\frac{13}{4} = 3\frac{1}{4}$, and $\frac{13}{5} = 2\frac{3}{5}$.
   Of the five answers given, the number that lies between 3 and 4 on a number line is $3\frac{1}{4}$ or $\frac{13}{4}$.

   Answer: (D)

7. Exactly 2 of the $2 + 3 + 4 = 9$ seeds are sunflower seeds.
   Therefore, the probability that Carrie chooses a sunflower seed is $\frac{2}{9}$.

   Answer: (A)

8. Since $x = 4$, then $y = 3 \times 4 = 12$.

   Answer: (A)

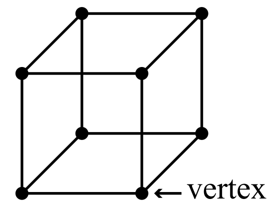
9. The sum of the three angles in any triangle is 180°.
   If one of the angles in an isosceles triangle measures 50°, then the sum of the measures of the two unknown angles in the triangle is $180^\circ - 50^\circ = 130^\circ$.
   Since the triangle is isosceles, then two of the angles in the triangle have equal measure.
   If the two unknown angles are equal in measure, then they each measure $130^\circ \div 2 = 65^\circ$.
   However, 65° and 65° is not one of the given answers.
   If the measure of one of the unknown angles is equal to the measure of the given angle, 50°, then the third angle in the triangle measures $180^\circ - 50^\circ - 50^\circ = 80^\circ$.
   Therefore, the measures of the other angles in this triangle could be 50° and 80°.

   Answer: (C)

10. Moving 3 letters clockwise from W, we arrive at the letter Z.
    Moving 1 letter clockwise from the letter Z, the alphabet begins again at A.
    Therefore, the letter that is 4 letters clockwise from W is A.
    Moving 4 letters clockwise from I, we arrive at the letter M.
    Moving 4 letters clockwise from N, we arrive at the letter R.
    The ciphertext of the message WIN is AMR.

    Answer: (C)

11. Every cube has exactly 8 vertices, as shown in the diagram.



ANSWER: (E)

12. The area of the 2 cm by 2 cm base of the rectangular prism is $2 \times 2 = 4$ cm^2.
The top face of the prism is identical to the base and so its area is also 4 cm^2.
Each of the 4 vertical faces of the prism has dimensions 2 cm by 1 cm, and thus has area $2 \times 1 = 2$ cm^2.
Therefore the surface area of the rectangular prism is $2 \times 4 + 4 \times 2 = 16$ cm^2.

ANSWER: (E)

13. Since 11 410 kg of rice is distributed into 3260 bags, then each bag contains $11\,410 \div 3260 = 3.5$ kg of rice.
Since a family uses 0.25 kg of rice each day, then it would take this family $3.5 \div 0.25 = 14$ days to use up one bag of rice.

ANSWER: (D)

14. Since Dalia's birthday is on a Wednesday, then any exact number of weeks after Dalia's birthday will also be a Wednesday.
Therefore, exactly 8 weeks after Dalia's birthday is also a Wednesday.
Since there are 7 days in each week, then $7 \times 8 = 56$ days after Dalia's birthday is a Wednesday.
Since 56 days after Dalia's birthday is a Wednesday, then 60 days after Dalia's birthday is a Sunday (since 4 days after Wednesday is Sunday).
Therefore, Bruce's birthday is on a Sunday.

ANSWER: (E)

15. *Solution 1*
Since each emu gets 2 treats and each chicken gets 4 treats, each of Karl's 30 birds gets *at least* 2 treats .
If Karl begins by giving his 30 birds exactly 2 treats each, then Karl will have given out $30 \times 2 = 60$ of the treats.
Since Karl has 100 treats to hand out, then he has $100 - 60 = 40$ treats left to give.
However, each emu has already received their 2 treats (since all 30 birds were given 2 treats).
So the remaining 40 treats must be given to chickens.
Each chicken is to receive 4 treats and has already received 2 treats.
Therefore, each chicken must receive 2 more treats.
Since there are 40 treats remaining, and each chicken receives 2 of these treats, then there are $40 \div 2 = 20$ chickens.
(We may check that if there are 20 chickens, then there are $30 - 20 = 10$ emus, and Karl would then give out $4 \times 20 + 2 \times 10 = 100$ treats.)

Solution 2
Using a trial and error approach, if Karl had 5 emus and $30 - 5 = 25$ chickens, then he would need to hand out $5 \times 2 + 25 \times 4 = 110$ treats.
Since Karl hands out 100 treats, we know that Karl has more emus than 5 (and fewer chickens than 25).
We show this attempt and continue with this approach in the table below.

Number of Emus	Number of Chickens	Number of Emu Treats	Number of Chicken Treats	Total Number of Treats
5	$30-5=25$	$5\times 2=10$	$25\times 4=100$	$10+100=110$
7	$30-7=23$	$7\times 2=14$	$23\times 4=92$	$14+92=106$
10	$30-10=20$	$10\times 2=20$	$20\times 4=80$	$20+80=100$

Therefore, Karl has 20 chickens.

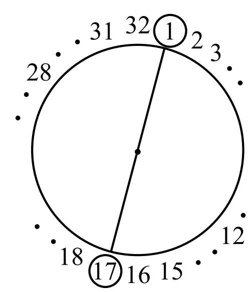
ANSWER: (D)

16. *Solution 1*

The integers 1 to 32 are spaced evenly and in order around the outside of a circle.
Consider drawing a first straight line that passes through the centre of the circle and joins any one pair of these 32 numbers.
This leaves $32-2=30$ numbers still to be paired.
Since this first line passes through the centre of the circle, it divides the circle in half.
In terms of the remaining 30 unpaired numbers, this means that 15 of these numbers lie on each side of the line drawn between the first pair.
Let the number that is paired with 12 be n.
If we draw the line through the centre joining 12 and n, then there are 15 numbers that lie between 12 and n (moving in either direction, clockwise or counter-clockwise).
Beginning at 12 and moving in the direction of the 13, the 15 numbers that lie between 12 and n are the numbers $13, 14, 15, \ldots, 26, 27$.
Therefore, the next number after 27 is the number n that is paired with 12.
The number paired with 12 is 28.

Solution 2

We begin by placing the integers 1 to 32, spaced evenly and in order, clockwise around the outside of a circle.
As in Solution 1, we recognize that there are 15 numbers on each side of the line which joins 1 with its partner.
Moving in a clockwise direction from 1, these 15 numbers are $2, 3, 4, \ldots, 15, 16$, and so 1 is paired with 17, as shown.
Since 2 is one number clockwise from 1, then the partner for 2 must be one number clockwise from 17, which is 18.
Similarly, 12 is 11 numbers clockwise from 1, so the partner for 12 must be 11 numbers clockwise from 17.
Therefore, the number paired with 12 is $17+11=28$.



ANSWER: (A)

17. We may begin by assuming that the area of the smallest circle is 1.

The area of the shaded middle ring is 6 times the area of the smallest circle, and thus has area 6.
The area of the unshaded outer ring is 12 times the area of the smallest circle, and thus has area 12.
The area of the largest circle is the sum of the areas of the smallest circle, the shaded middle ring, and the unshaded outer ring, or $1+6+12=19$.
Therefore, the area of the smallest circle is $\frac{1}{19}$ of the area of the largest circle.
Note: We assumed the area of the smallest circle was 1, however we could have assumed it to have any area. For example, assume the area of the smallest circle is 5 and redo the question. What is your final answer?

ANSWER: (E)

18. For the product of two integers to equal 1, the two integers must both equal 1 or must both equal -1.
Similarly, if the product of six integers is equal to 1, then each of the six integers must equal 1 or -1.
For the product of six integers, each of which is equal to 1 or -1, to equal 1, the number of -1s must be even, because an odd number of -1s would give a product that is negative.
That is, there must be zero, two, four or six -1s among the six integers.
We summarize these four possibilities in the table below.

Number of -1s	Product of the six integers	Sum of the six integers
0	$(1)(1)(1)(1)(1)(1) = 1$	$1+1+1+1+1+1=6$
2	$(-1)(-1)(1)(1)(1)(1) = 1$	$(-1)+(-1)+1+1+1+1=2$
4	$(-1)(-1)(-1)(-1)(1)(1) = 1$	$(-1)+(-1)+(-1)+(-1)+1+1=-2$
6	$(-1)(-1)(-1)(-1)(-1)(-1) = 1$	$(-1)+(-1)+(-1)+(-1)+(-1)+(-1)=-6$

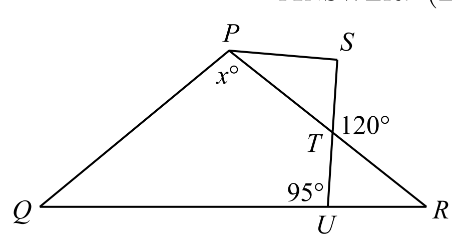
Of the answers given, the sum of such a group of six integers cannot equal 0.

Answer: (C)

19. Since the heights of the 4 athletes on the team are all different, then if Laurissa's height is different than each of these, there is no single mode height.
Therefore, Laurissa's height must be equal to the height of one of the 4 athletes on the team for there to be a single mode.
If Laurissa's height is 135 cm, then the median height of the 5 athletes is 160 cm which is not possible, since the median does not equal the mode.
Similarly, if Laurissa's height is 175 cm, then the median height of the 5 athletes is 170 cm which is not possible.
Therefore, Laurissa's height must equal 160 cm or 170 cm, since in either case the median height of the 5 athletes will equal Laurissa's height, which is the mode.
If Laurissa's height is 170 cm, then the mean height of the 5 athletes is
$\dfrac{135+160+170+170+175}{5} = 162$ cm.
If Laurissa's height is 160 cm, then the mean height of the 5 athletes is
$\dfrac{135+160+160+170+175}{5} = 160$ cm.
When Laurissa's height is 160 cm, the heights of the 5 athletes (measured in cm) are: 135, 160, 160, 170, 175.
In this case, each of the mode, median and mean height of the 5 athletes equals 160 cm.

Answer: (B)

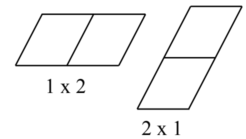
20. We begin by labelling points S, T and U, as shown.



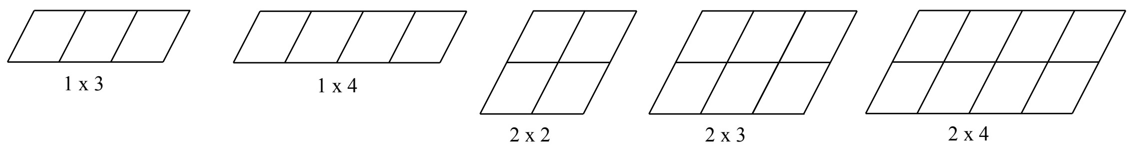
Since S, T, U lie on a straight line, $\angle STU$ measures 180°.
Therefore, $\angle RTU = 180^\circ - \angle STR = 180^\circ - 120^\circ = 60^\circ$.
Similarly, Q, U, R lie on a straight line, and so $\angle QUR$ measures 180°.
Therefore, $\angle TUR = 180^\circ - \angle TUQ = 180^\circ - 95^\circ = 85^\circ$.
The sum of the angles in $\triangle TUR$ is 180°.
Thus, $\angle TRU = 180^\circ - \angle RTU - \angle TUR = 180^\circ - 60^\circ - 85^\circ = 35^\circ$.
Since $\triangle PQR$ is isosceles with $PQ = PR$, then $\angle PQR = \angle PRQ = 35^\circ$.
Finally, the sum of the angles in $\triangle PQR$ is 180°, and so $x^\circ = 180^\circ - \angle PQR - \angle PRQ$ or $x^\circ = 180^\circ - 35^\circ - 35^\circ$ and so $x = 110$.

Answer: (A)

21. The figure formed by combining a pair of adjacent small parallelograms, is also a parallelogram.
For example, each of the two figures shown is a parallelogram.
The reason for this is that opposite sides of these new figures are equal in length and they are parallel. We use the notation $a \times b$ to mean that the new figure has a rows of the small parallelograms and b columns of the small parallelograms.



Similarly, more than 2 small parallelograms can be combined to form new parallelograms.
In addition to the small parallelogram (1×1) and the 1×2 and 2×1 parallelograms shown above, the sizes of the remaining parallelograms that appear in the figure are shown below.



In the table below, the number of parallelograms of each of the different sizes is shown.

Size	1×1	1×2	2×1	1×3	1×4	2×2	2×3	2×4
Number of Parallelograms	8	6	4	4	2	3	2	1

The number of parallelograms appearing in the figure is $8 + 6 + 4 + 4 + 2 + 3 + 2 + 1 = 30$.

ANSWER: (B)

22. *Solution 1*
The number of dimes in the jar is one more than the number of nickels.
If we remove one dime from the jar, then the number of coins remaining in the jar is $50 - 1 = 49$, and the value of the coins remaining in the jar is $\$5.00 - \$0.10 = \$4.90$.
Also, the number of dimes remaining in the jar is now equal to the number of nickels remaining in the jar, and the number of nickels remaining in the jar is three times the number of quarters remaining in the jar.
That is, for every 1 quarter remaining in the jar, there are 3 nickels and 3 dimes.
Consider groups consisting of exactly 1 quarter, 3 nickels and 3 dimes.
In each of these groups, there are 7 coins whose total value is $\$0.25 + 3 \times \$0.05 + 3 \times \$0.10 = \$0.25 + \$0.15 + \$0.30 = \$0.70$.
Since there are 49 coins having a value of \$4.90 remaining in the jar, then there must be 7 such groups of 7 coins remaining in the jar (since $7 \times 7 = 49$).
(We may check that 7 such groups of coins, with each group having a value of \$0.70, has a total value of $7 \times \$0.70 = \$4.90$, as required.)
Therefore, there are 7 quarters in the jar.

Solution 2
To find the number of quarters in the jar, we need only focus on the total number of coins in the jar, 50, or on the total value of the coins in the jar, \$5.00.
In the solution that follows, we consider both the number of coins in the jar as well as the value of the coins in the jar, to demonstrate that each approach leads to the same answer.
We use a trial and error approach.
Suppose that the number of quarters in the jar is 5 (the smallest of the possible answers given).
The value of 5 quarters is $5 \times 25¢ = 125¢$.
Since the number of nickels in the jar is three times the number of quarters, there would be $3 \times 5 = 15$ nickels in the jar.
The value of 15 nickels is $15 \times 5¢ = 75¢$.

Since the number of dimes in the jar is one more than the number of nickels, there would be $15 + 1 = 16$ dimes in the jar.
The value of 16 dimes is $16 \times 10¢ = 160¢$.
If there were 5 quarters in the jar, then the total number of coins in the jar would be $5 + 15 + 16 = 36$, and so there must be more than 5 quarters in the jar.
Similarly, if there were 5 quarters in the jar, then the total value of the coins in the jar would be $125¢ + 75¢ + 160¢ = 360¢$.
Since the value of the coins in the jar is \$5.00 or 500¢ then the number of quarters in the jar is greater than 5.
We summarize our next two trials in the table below.

Number of Quarters	Value of Quarters	Number of Nickels	Value of Nickels	Number of Dimes	Value of Dimes	Total Value of Coins
6	150¢	18	90¢	19	190¢	430¢
7	175¢	21	105¢	22	220¢	500¢

When there are 7 quarters in the jar, there are $7 + 21 + 22 = 50$ coins in the jar, as required.
When there are 7 quarters in the jar, the value of the coins in the jar is $175¢ + 105¢ + 220¢ = 500¢$ or \$5.00, as required.
In either case, the number of quarters in the jar is 7.

Answer: (A)

23. In each block $1223334444 \cdots 999999999$, there is 1 digit 1, 2 digits 2, 3 digits 3, and so on.
The total number of digits written in each block is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$.
We note that $1953 \div 45$ gives a quotient of 43 and a remainder of 18 (that is, $1953 = 45 \times 43 + 18$).
Since each block contains 45 digits, then 43 blocks contain $43 \times 45 = 1935$ digits.
Since $1953 - 1935 = 18$, then the 18^{th} digit written in the next block (the 44^{th} block) will be the 1953^{rd} digit written.
Writing out the first 18 digits in a block, we get 122333444455555666, and so the 1953^{rd} digit written is a 6.

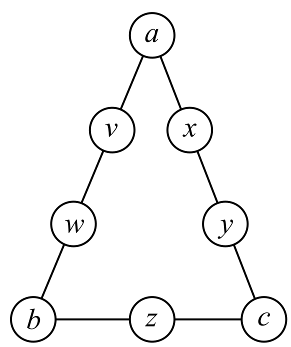
Answer: (C)

24. For a positive integer to be divisible by 9, the sum of its digits must be divisible by 9.
In this problem, we want to count the number of six-digit positive integers containing 2018 and divisible by 9.
Thus, we must find the remaining two digits, which together with 2018, form a six-digit positive integer that is divisible by 9.
The digits 2018 have a sum of $2 + 0 + 1 + 8 = 11$.
Let the remaining two digits be a and b so that the six-digit positive integer is $ab2018$ or $ba2018$ or $a2018b$ or $b2018a$ or $2018ab$ or $2018ba$.
When the digits a and b are added to 11, the sum must be divisible by 9.
That is, the sum $a + b + 11$ must be divisible by 9.
The smallest that each of a and b can be is 0, and so the smallest that the sum $a + b + 11$ can be is $0 + 0 + 11 = 11$.
The largest that each of a and b can be is 9, and so the largest that the sum $a + b + 11$ can be is $9 + 9 + 11 = 29$.
The only integers between 11 and 29 that are divisible by 9 are 18 and 27.
Therefore, either $a + b = 18 - 11 = 7$ or $a + b = 27 - 11 = 16$.
If $a + b = 7$, then the digits a and b are 0 and 7 or 1 and 6 or 2 and 5 or 3 and 4, in some order.
If a and b are 1 and 6, then the possible six-digit integers are $162\,018, 612\,018, 120\,186, 620\,181,$

201 816, and 201 861.
In this case, there are 6 possible six-digit positive integers.
Similarly, if a and b are 2 and 5, then there are 6 possible six-digit integers.
Likewise, if a and b are 3 and 4, then there are 6 possible six-digit integers.
If a and b are 0 and 7, then the possible six-digit integers are $702\,018, 720\,180, 201\,870$, and $201\,807$, since the integer cannot begin with the digit 0.
In this case, there are 4 possible six-digit positive integers.
Therefore, for the case in which the sum of a and b is 7, there are $6 + 6 + 6 + 4 = 22$ possible six-digit integers.
Finally, we consider the case for which the sum of the digits a and b is 16.
If $a + b = 16$, then the digits a and b are 7 and 9, or 8 and 8.
If a and b are 7 and 9, then there are again 6 possible six-digit integers ($792\,018, 972\,018$, $720\,1869, 920\,187, 201\,879$, and $201\,897$).
If a and b are 8 and 8, then there are 3 possible six-digit integers: $882\,018, 820\,188$, and $201\,888$.
Therefore, for the case in which the sum of a and b is 16, there are $6 + 3 = 9$ possible six-digit integers, and so there are $22 + 9 = 31$ six-digit positive integers in total.
We note that all of these 31 six-digit positive integers are different from one another, and that they are the only six-digit positive integers satisfying the given conditions.
Therefore, there are 31 six-digit positive integers that are divisible by 9 and that contain the digits 2018 together and in this order.

Answer: (C)

25. We label the unknown numbers in the circles as shown:



Since the sum of the numbers along each side of the triangle is S then

$$S = a + v + w + b \qquad S = a + x + y + c \qquad S = b + z + c$$

When we add the numbers along each of the three sides of the triangles, we include each of a, b and c twice and obtain

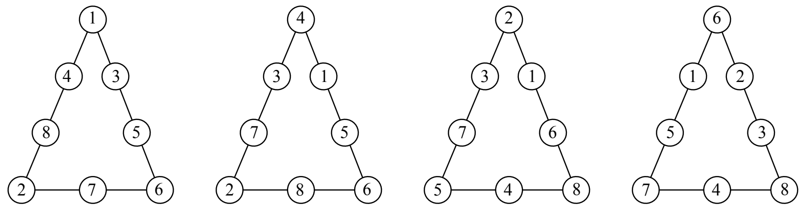
$$S+S+S = (a+v+w+b)+(a+x+y+c)+(b+z+c) = (a+v+w+b+z+c+y+x)+a+b+c$$

Now the numbers a, v, w, b, z, c, y, x are the numbers $1, 2, 3, 4, 5, 6, 7, 8$ in some order.
This means that $a + v + w + b + z + c + y + x = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$.
Therefore,

$$3S = 36 + a + b + c$$

Since $3S$ is a multiple of 3 and 36 is a multiple of 3, then $a + b + c$ (which equals $3S - 36$) must also be a multiple of 3.
Looking at the possible numbers that can go in the circles, the smallest that $a + b + c$ can be is

$1 + 2 + 3$ or 6, which would make $3S = 36 + 6 = 42$ or $S = 14$. S cannot be any smaller than 14 because $a + b + c$ cannnot be any smaller than 6 and so $3S$ cannot be any smaller than 42. Looking at the possible numbers that can go in the circles, the largest that $a + b + c$ can be is $6 + 7 + 8$ or 21, which would make $3S = 36 + 21 = 57$ or $S = 19$. S cannot be any larger than 19 because $a + b + c$ cannot be any larger than 21 and so $3S$ cannot be any larger than 57.
So which of the values $S = 14, 15, 16, 17, 18, 19$ is actually possible?
The following diagrams show ways of completing the triangle with $S = 15, 16, 17, 19$:



Coming up with these examples requires a combination of reasoning and fiddling.
For example, consider the case when $S = 15$.
Since $3S = 36 + a + b + c$ and $S = 15$, then $a + b + c = 3 \times 15 - 36 = 9$.
In the given example, we have $a = 1$, $b = 2$ and $c = 6$.
Since the bottom row $(b + z + c)$ has the smallest number of circles, we put the largest of a, b, c here ($b = 2$ and $c = 6$) and then set $z = 15 - b - c = 7$. A bit of fiddling allows us to choose u, v, x, y appropriately to get the desired sums on the two other sides.
We note that there are other possible combinations of a, b, c with $a + b + c = 9$ (namely, $1, 3, 5$ and $2, 3, 4$). It turns out that neither of these possibilities can produce a triangle with $S = 15$.
In a similar way, we can determine examples like those shown with $S = 16, 17, 19$.

To complete the solution, we show that $S = 14$ and $S = 18$ are not possible.
Suppose that $S = 14$.
In this case, $a + b + c = 3S - 36 = 3 \times 14 - 36 = 6$.
The only integers from the list $1, 2, 3, 4, 5, 6, 7, 8$ which give this sum are $1, 2, 3$.
Consider the bottom row, which should have $b + z + c = 14$.
Since a, b, c are $1, 2, 3$ in some order, then $b + c$ is at most $2 + 3 = 5$.
Since the maximum number in the triangle is 8, then z is at most 8.
This makes $b + z + c$ at most $5 + 8 = 13$, which means that $b + z + c$ cannot equal 14.
This means that we cannot build a triangle with $S = 14$.
Suppose that $S = 18$.
In this case, $a + b + c = 3S - 36 = 3 \times 18 - 36 = 18$.
There are several possible sets of values for a, b, c: $3, 7, 8$ and $4, 6, 8$ and $5, 6, 7$.
Consider the bottom row again, which should have $b + z + c = 18$.
Here we have $a + b + c = 18$ and $b + z + c = 18$.
Since b and c are common to these sums and the total is the same in each case, then $a = z$, which is not allowed.
This means that we cannot build a triangle with $S = 18$.

In summary, the possible values of S are $15, 16, 17, 19$.
The sum of these values is 67.

Answer: (E)

Grade 8

1. Since the cost of 1 melon is \$3, then the cost of 6 melons is $6 \times \$3 = \$18$.

   Answer: (C)

2. The number line shown has length $1 - 0 = 1$.
   The number line is divided into 10 equal parts, and so each part has length $1 \div 10 = 0.1$.
   The P is positioned 2 of these equal parts before 1, and so the value of P is $1 - (2 \times 0.1) = 1 - 0.2 = 0.8$.
   (Similarly, we could note that P is positioned 8 equal parts after 0, and so the value of P is $8 \times 0.1 = 0.8$.)

   Answer: (D)

3. Following the correct order of operations, we get $(2+3)^2-(2^2+3^2) = 5^2-(4+9) = 25-13 = 12$.

   Answer: (B)

4. Since Lakshmi is travelling at 50 km each hour, then in one half hour (30 minutes) she will travel $50 \div 2 = 25$ km.

   Answer: (C)

5. Exactly 2 of the $3 + 2 + 4 + 6 = 15$ flowers are tulips.
   Therefore, the probability that Evgeny randomly chooses a tulip is $\frac{2}{15}$.

   Answer: (E)

6. The range of the students' heights is equal to the difference between the height of the tallest student and the height of the shortest student.
   Reading from the graph, Emma is the tallest student and her height is approximately 175 cm.
   Kinley is the shortest student and her height is approximately 100 cm.
   Therefore, the range of heights is closest to $175 - 100 = 75$ cm.

   Answer: (A)

7. *Solution 1*
   The circumference of a circle, C, is given by the formula $C = \pi \times d$, where d is the diameter of the circle.
   Since the circle has a diameter of 1 cm, then its circumference is $C = \pi \times 1 = \pi$ cm.
   Since π is approximately 3.14, then the circumference of the circle is between 3 cm and 4 cm.

   Solution 2
   The circumference of a circle, C, is given by the formula $C = 2 \times \pi \times r$, where r is the radius of the circle.
   Since the circle has a diameter of 1 cm, then its radius is $r = \frac{1}{2}$ cm, and so the circumference is $C = 2 \times \pi \times \frac{1}{2} = \pi$ cm.
   Since π is approximately 3.14, then the circumference of the circle is between 3 cm and 4 cm.

   Answer: (B)

8. The ratio of the amount of cake eaten by Rich to the amount of cake eaten by Ben is $3 : 1$.
   Thus, if the cake was divided into 4 pieces of equal size, then Rich ate 3 pieces and Ben ate 1 piece or Ben ate $\frac{1}{4}$ of the cake.
   Converting to a percent, Ben ate $\frac{1}{4} \times 100\% = 0.25 \times 100\% = 25\%$ of the cake.

   Answer: (D)

9. Moving 3 letters clockwise from W, we arrive at the letter Z.
Moving 1 letter clockwise from the letter Z, the alphabet begins again at A.
Therefore, the letter that is 4 letters clockwise from W is A.
Moving 4 letters clockwise from I, we arrive at the letter M.
Moving 4 letters clockwise from N, we arrive at the letter R.
The ciphertext of the message WIN is AMR.

ANSWER: (C)

10. The smallest of 3 consecutive even numbers is 2 less than the middle number.
The largest of 3 consecutive even numbers is 2 more than the middle number.
Therefore, the sum of 3 consecutive even numbers is three times the middle number.
To see this, consider subtracting 2 from the largest of the 3 numbers, and adding 2 to the smallest of the 3 numbers.
Since we have subtracted 2 and also added 2, then the sum of these 3 numbers is equal to the sum of the original 3 numbers.
However, if we subtract 2 from the largest number, the result is equal to the middle number, and if we add 2 to the smallest number, the result is equal to the middle number.
Therefore, the sum of any 3 consecutive even numbers is equal to three times the middle number.
Since the sum of the 3 consecutive even numbers is 312, then the middle number is equal to $312 \div 3 = 104$.
If the middle number is 104, then the largest of the 3 consecutive even numbers is $104+2 = 106$.
(We may check that $102 + 104 + 106$ is indeed equal to 312.)

ANSWER: (B)

11. If $4x + 12 = 48$, then $4x = 48 - 12$ or $4x = 36$, and so $x = \frac{36}{4} = 9$.

ANSWER: (E)

12. The time in Vancouver is 3 hours earlier than the time in Toronto.
Therefore, when it is 6:30 p.m. in Toronto, the time in Vancouver is 3:30 p.m..

ANSWER: (C)

13. *Solution 1*
Mateo receives \$20 every hour for one week.
Since there are 24 hours in each day, and 7 days in each week, then Mateo receives $\$20 \times 24 \times 7 = \$3360$ over the one week period.
Sydney receives \$400 every day for one week.
Since there are 7 days in each week, then Sydney receives $\$400 \times 7 = \$2800$ over the one week period.
The difference in the total amounts of money that they receive over the one week period is $\$3360 - \$2800 = \$560$.

Solution 2
Mateo receives \$20 every hour for one week.
Since there are 24 hours in each day, then Mateo receives $\$20 \times 24 = \$480$ each day.
Sydney receives \$400 each day, and so Mateo receives $\$480 - \$400 = \$80$ more than Sydney each day.
Since there are 7 days in one week, then the difference in the total amounts of money that they receive over the one week period is $\$80 \times 7 = \$560$.

ANSWER: (A)

14. Since $2018 = 2 \times 1009$, and both 2 and 1009 are prime numbers, then the required sum is $2 + 1009 = 1011$.
Note: In the question, we are given that 2018 has exactly two divisors that are prime numbers, and since 2 is a prime divisor of 2018, then 1009 must be the other prime divisor.

Answer: (B)

15. The first place award can be given out to any one of the 5 classmates.
Once the first place award has been given, there are 4 classmates remaining who could be awarded second place (since the classmate who was awarded first place cannot also be awarded second place).
For each of the 5 possible first place winners, there are 4 classmates who could be awarded second place, and so there are 5×4 ways that the first and second place awards can be given out.
Once the first and second place awards have been given, there are 3 classmates remaining who could be awarded the third place award (since the classmates who were awarded first place and second place cannot also be awarded third place).
For each of the 5 possible first place winners, there are 4 classmates who could be awarded second place, and there are 3 classmates who could be awarded third place.
That is, there are $5 \times 4 \times 3 = 60$ ways that the first, second and third place awards can be given out.

Answer: (B)

16. For the product of two integers to equal 1, the two integers must both equal 1 or must both equal -1.
Similarly, if the product of six integers is equal to 1, then each of the six integers must equal 1 or -1.
For the product of six integers, each of which is equal to 1 or -1, to equal 1, the number of -1s must be even, because an odd number of -1s would give a product that is negative.
That is, there must be zero, two, four or six -1s among the six integers.
We summarize these four possibilities in the table below.

Number of -1s	Product of the six integers	Sum of the six integers
0	$(1)(1)(1)(1)(1)(1) = 1$	$1+1+1+1+1+1 = 6$
2	$(-1)(-1)(1)(1)(1)(1) = 1$	$(-1)+(-1)+1+1+1+1 = 2$
4	$(-1)(-1)(-1)(-1)(1)(1) = 1$	$(-1)+(-1)+(-1)+(-1)+1+1 = -2$
6	$(-1)(-1)(-1)(-1)(-1)(-1) = 1$	$(-1)+(-1)+(-1)+(-1)+(-1)+(-1) = -6$

Of the answers given, the sum of such a group of six integers cannot equal 0.

Answer: (C)

17. *Solution 1*
Each translation to the right 5 units increases the x-coordinate of point A by 5.
Similarly, each translation up 3 units increases the y-coordinate of point A by 3.
After 1 translation, the point $A(-3, 2)$ would be at $B(-3 + 5, 2 + 3)$ or $B(2, 5)$.
After 2 translations, the point $A(-3, 2)$ would be at $C(2 + 5, 5 + 3)$ or $C(7, 8)$.
After 3 translations, the point $A(-3, 2)$ would be at $D(7 + 5, 8 + 3)$ or $D(12, 11)$.
After 4 translations, the point $A(-3, 2)$ would be at $E(12 + 5, 11 + 3)$ or $E(17, 14)$.
After 5 translations, the point $A(-3, 2)$ would be at $F(17 + 5, 14 + 3)$ or $F(22, 17)$.
After 6 translations, the point $A(-3, 2)$ would be at $G(22 + 5, 17 + 3)$ or $G(27, 20)$.
After these 6 translations, the point is at $(27, 20)$ and so $x + y = 27 + 20 = 47$.

Solution 2

Each translation to the right 5 units increases the x-coordinate of point A by 5.
Similarly, each translation up 3 units increases the y-coordinate of point A by 3.
Therefore, each translation of point $A(-3,2)$ to the right 5 units and up 3 units increases the sum of the x- and y-coordinates, $x+y$, by $5+3=8$.
After 6 of these translations, the sum $x+y$ will increase by $6\times 8=48$.
The sum of the x- and y-coordinates of point $A(-3,2)$ is $-3+2=-1$.
After these 6 translations, the value of $x+y$ is $-1+48=47$.

ANSWER: (D)

18. *Solution 1*

The volume of any rectangular prism is given by the product of the length, the width, and the height of the prism.
When the length of the prism is doubled, the product of the new length, the width, and the height of the prism doubles, and so the volume of the prism doubles.
Since the original prism has a volume of 30 cm^3, then doubling the length creates a new prism with volume $30\times 2=60$ cm^3.
When the width of this new prism is tripled, the product of the length, the new width, and the height of the prism is tripled, and so the volume of the prism triples.
Since the prism has a volume of 60 cm^3, then tripling the width creates a new prism with volume $60\times 3=180$ cm^3.
When the height of the prism is divided by four, the product of the length, the width, and the new height of the prism is divided by four, and so the volume of the prism is divided by four.
Since the prism has a volume of 180 cm^3, then dividing the height by four creates a new prism with volume $180\div 4=45$ cm^3.

Solution 2

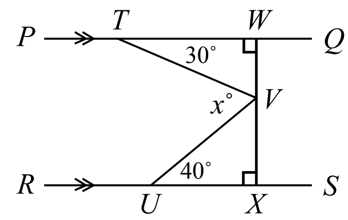
The volume of any rectangular prism is given by the product of its length, l, its width, w, and its height, h, which equals lwh.
When the length of the prism is doubled, the length of the new prism is $2l$.
Similarly, when the width is tripled, the new width is $3w$, and when the height is divided by four, the new height is $\frac{1}{4}h$.
Therefore, the volume of the new prism is the product of its length, $2l$, its width, $3w$, and its height, $\frac{1}{4}h$, which equals $(2l)(3w)(\frac{1}{4}h)$ or $\frac{3}{2}lwh$.
That is, the volume of the new prism is $\frac{3}{2}$ times larger than the volume of the original prism.
Since the original prism has a volume of 30 cm^3, then doubling the length, tripling the width, and dividing the height by four, creates a new prism with volume $30\times\frac{3}{2}=\frac{90}{2}=45$ cm^3.

ANSWER: (E)

19. The mean height of the group of children is equal to the sum of the heights of the children divided by the number of children in the group.
Therefore, the mean height of the group of children increases by 6 cm if the sum of the increases in the heights of the children, divided by the number of children in the group, is equal to 6.
If 12 of the children were each 8 cm taller, then the sum of the increases in the heights of the children would be $12\times 8=96$ cm.
Thus, 96 divided by the number of children in the group is equal to 6.
Since $96\div 16=6$, then the number of children in the group is 16.

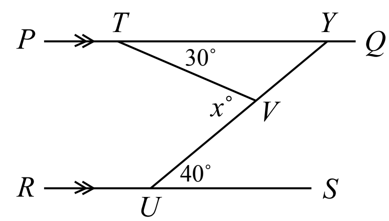
ANSWER: (A)

20. *Solution 1*



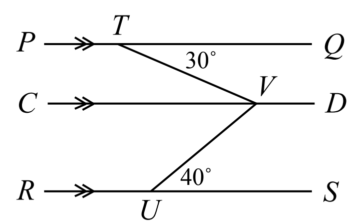
We begin by constructing a line segment WX perpendicular to PQ and passing through V, as shown.
Since RS is parallel to PQ, then WX is also perpendicular to RS.
In $\triangle TWV$, $\angle TWV = 90^\circ$ and $\angle WTV = 30^\circ$.
Since the sum of the angles in a triangle is 180°, then $\angle TVW = 180^\circ - 30^\circ - 90^\circ = 60^\circ$.
In $\triangle UXV$, $\angle UXV = 90^\circ$ and $\angle VUX = 40^\circ$.
Similarly, $\angle UVX = 180^\circ - 40^\circ - 90^\circ = 50^\circ$.
Since WX is a straight line segment, then $\angle TVW + \angle TVU + \angle UVX = 180^\circ$.
That is, $60^\circ + \angle TVU + 50^\circ = 180^\circ$ or $\angle TVU = 180^\circ - 60^\circ - 50^\circ$ or $\angle TVU = 70^\circ$, and so $x = 70$.

Solution 2



We begin by extending line segment UV to meet PQ at Y, as shown.
Since RS is parallel to PQ, then $\angle TYV$ and $\angle VUS$ are alternate angles, and so $\angle TYV = \angle VUS = 40^\circ$.
In $\triangle TYV$, $\angle TYV = 40^\circ$ and $\angle YTV = 30^\circ$.
Since the sum of the angles in a triangle is 180°, then $\angle TVY = 180^\circ - 40^\circ - 30^\circ = 110^\circ$.
Since UY is a straight line segment, then $\angle TVU + \angle TVY = 180^\circ$.
That is, $\angle TVU + 110^\circ = 180^\circ$ or $\angle TVU = 180^\circ - 110^\circ$ or $\angle TVU = 70^\circ$, and so $x = 70$.

Solution 3



We begin by constructing a line segment CD parallel to both PQ and RS, and passing through V, as shown.
Since CD is parallel to PQ, then $\angle QTV$ and $\angle TVC$ are alternate angles, and so $\angle TVC = \angle QTV = 30^\circ$.
Similarly, since CD is parallel to RS, then $\angle CVU$ and $\angle VUS$ are alternate angles, and so $\angle CVU = \angle VUS = 40^\circ$.
Since $\angle TVU = \angle TVC + \angle CVU$, then $\angle TVU = 30^\circ + 40^\circ = 70^\circ$, and so $x = 70$.

ANSWER: (D)

21. *Solution 1*

We begin by assuming that there are 100 marbles in the bag.
The probability of choosing a brown marble is 0.3, and so the number of brown marbles in the bag must be 30 since $\frac{30}{100} = 0.3$.
Choosing a brown marble is three times as likely as choosing a purple marble, and so the number of purple marbles in the bag must be $30 \div 3 = 10$.
Choosing a green marble is equally likely as choosing a purple marble, and so there must also be 10 green marbles in the bag.
Since there are 30 brown marbles, 10 purple marbles, and 10 green marbles in the bag, then there are $100 - 30 - 10 - 10 = 50$ marbles in the bag that are either red or yellow.
Choosing a red marble is equally likely as choosing a yellow marble, and so the number of red marbles in the bag must equal the number of yellow marbles in the bag.
Therefore, the number of red marbles in the bag is $50 \div 2 = 25$.
Of the 100 marbles in the bag, there are $25 + 10 = 35$ marbles that are either red or green.
The probability of choosing a marble that is either red or green is $\frac{35}{100} = 0.35$.

Solution 2

The probability of choosing a brown marble is 0.3.

The probability of choosing a brown marble is three times that of choosing a purple marble, and so the probability of choosing a purple marble is $0.3 \div 3 = 0.1$.

The probability of choosing a green marble is equal to that of choosing a purple marble, and so the probability of choosing a green marble is also 0.1.

Let the probability of choosing a red marble be p.

The probability of choosing a red marble is equal to that of choosing a yellow marble, and so the probability of choosing a yellow marble is also p.

The total of the probabilities of choosing a marble must be 1.

Therefore, $0.3 + 0.1 + 0.1 + p + p = 1$ or $0.5 + 2p = 1$ or $2p = 0.5$, and so $p = 0.5 \div 2 = 0.25$.

The probability of choosing a red marble is 0.25 and the probability of choosing a green marble is 0.1, and so the probability of choosing a marble that is either red or green is $0.25+0.1 = 0.35$.

Answer: (C)

22. The area of square $PQRS$ is $(30)(30) = 900$.

Each of the 5 regions has equal area, and so the area of each region is $900 \div 5 = 180$.

The area of $\triangle SPT$ is equal to $\frac{1}{2}(PS)(PT) = \frac{1}{2}(30)(PT) = 15(PT)$.

The area of $\triangle SPT$ is 180, and so $15(PT) = 180$ or $PT = 180 \div 15 = 12$.

The area of $\triangle STU$ is 180.

Let the base of $\triangle STU$ be UT.

The height of $\triangle STU$ is equal to PS since PS is the perpendicular distance between base UT (extended) and the vertex S.

The area of $\triangle STU$ is equal to $\frac{1}{2}(PS)(UT) = \frac{1}{2}(30)(UT) = 15(UT)$.

The area of $\triangle STU$ is 180, and so $15(UT) = 180$ or $UT = 180 \div 15 = 12$.

In $\triangle SPT$, $\angle SPT = 90^\circ$. By the Pythagorean Theorem, $ST^2 = PS^2 + PT^2$ or $ST^2 = 30^2 + 12^2$ or $ST^2 = 900 + 144 = 1044$, and so $ST = \sqrt{1044}$ (since $ST > 0$).

In $\triangle SPU$, $\angle SPU = 90^\circ$ and $PU = PT + UT = 12 + 12 = 24$.

By the Pythagorean Theorem, $SU^2 = PS^2+PU^2$ or $ST^2 = 30^2+24^2$ or $SU^2 = 900+576 = 1476$, and so $SU = \sqrt{1476}$ (since $SU > 0$).

Therefore, $\dfrac{SU}{ST} = \dfrac{\sqrt{1476}}{\sqrt{1044}}$ which is approximately equal to 1.189.

Of the answers given, $\dfrac{SU}{ST}$ is closest to 1.19.

Answer: (B)

23. *Solution 1*

In the table, we determine the value of the product $n(n + 1)(n + 2)$ for the first 10 positive integers:

n	$n(n+1)(n+2)$
1	$1 \times 2 \times 3 = 6$
2	$2 \times 3 \times 4 = 24$
3	$3 \times 4 \times 5 = 60$
4	$4 \times 5 \times 6 = 120$
5	$5 \times 6 \times 7 = 210$
6	$6 \times 7 \times 8 = 336$
7	$7 \times 8 \times 9 = 504$
8	$8 \times 9 \times 10 = 720$
9	$9 \times 10 \times 11 = 990$
10	$10 \times 11 \times 12 = 1320$

From the table, we see that $n(n+1)(n+2)$ is a multiple of 5 when $n = 3, 4, 5, 8, 9, 10$.
In general, because 5 is a prime number, the product $n(n+1)(n+2)$ is a multiple of 5 exactly when at least one of its factors $n, n+1, n+2$ is a multiple of 5.
A positive integer is a multiple of 5 when its units (ones) digit is either 0 or 5.
Next, we make a table that lists the units digits of $n+1$ and $n+2$ depending on the units digit of n:

Units digit of n	Units digit of $n+1$	Units digit of $n+2$
1	2	3
2	3	4
3	4	5
4	5	6
5	6	7
6	7	8
7	8	9
8	9	0
9	0	1
0	1	2

From the table, one of the three factors has a units digit of 0 or 5 exactly when the units digit of n is one of $3, 4, 5, 8, 9, 0$. (Notice that this agrees with the first table above.)
This means that 6 out of each block of 10 values of n ending at a multiple of 10 give a value for $n(n+1)(n+2)$ that is a multiple of 5.
We are asked for the 2018^{th} positive integer n for which $n(n+1)(n+2)$ is a multiple of 5.
Note that $2018 = 336 \times 6 + 2$.
This means that, in the first $336 \times 10 = 3360$ positive integers, there are $336 \times 6 = 2016$ integers n for which $n(n+1)(n+2)$ is a multiple of 5. (Six out of every ten integers have this property.)
We need to count two more integers along the list.
The next two integers n for which $n(n+1)(n+2)$ is a multiple of 5 will have units digits 3 and 4, and so are 3363 and 3364.
This means that 3364 is the 2018^{th} integer with this property.

Solution 2
In the table below, we determine the value of the product $n(n+1)(n+2)$ for the first 10 positive integers n.

n	$n(n+1)(n+2)$
1	$1 \times 2 \times 3 = 6$
2	$2 \times 3 \times 4 = 24$
3	$3 \times 4 \times 5 = 60$
4	$4 \times 5 \times 6 = 120$
5	$5 \times 6 \times 7 = 210$
6	$6 \times 7 \times 8 = 336$
7	$7 \times 8 \times 9 = 504$
8	$8 \times 9 \times 10 = 720$
9	$9 \times 10 \times 11 = 990$
10	$10 \times 11 \times 12 = 1320$

From the table, we see that the value of $n(n+1)(n+2)$ is not a multiple of 5 when $n = 1$ or when $n = 2$, but that $n(n+1)(n+2)$ is a multiple of 5 when $n = 3, 4, 5$.

Similarly, we see that the value of $n(n+1)(n+2)$ is a not multiple of 5 when $n = 6, 7$, but that $n(n + 1)(n + 2)$ is a multiple of 5 when $n = 8, 9, 10$.
That is, if we consider groups of 5 consecutive integers beginning at $n = 1$, it appears that for the first 2 integers in the group, the value of $n(n + 1)(n + 2)$ is not a multiple of 5, and for the last 3 integers in the group, the value of $n(n + 1)(n + 2)$ is a multiple of 5.
Will this pattern continue?
Since 5 is a prime number, then for each value of $n(n + 1)(n + 2)$ that is a multiple of 5, at least one of the factors $n, n + 1$ or $n + 2$ must be divisible by 5.
(We also note that for each value of $n(n+1)(n+2)$ that is not a multiple of 5, each of n, $n+1$ and $n + 2$ is not divisible by 5.)
For what values of n is at least one of n, $n+1$ or $n+2$ divisible by 5, and thus $n(n+1)(n+2)$ divisible by 5?
When n is a multiple of 5, then the value of $n(n + 1)(n + 2)$ is divisible by 5.
When n is one 1 less than a multiple of 5, then $n+1$ is a multiple of 5 and and so $n(n+1)(n+2)$ is divisible by 5.
Finally, when n is 2 less than a multiple of 5, then $n + 2$ is a multiple of 5 and and so $n(n + 1)(n + 2)$ is divisible by 5.
We also note that when n is 3 less than a multiple of 5, each of n, $n + 1$ (which is 2 less than a multiple of 5), and $n + 2$ (which is 1 less than a multiple of 5) is not divisible by 5, and so $n(n + 1)(n + 2)$ is not divisible by 5.
Similarly, when n is 4 less than a multiple of 5, then each of n, $n + 1$ (which is 3 less than a multiple of 5), and $n + 2$ (which is 2 less than a multiple of 5) is not divisible by 5, and so $n(n + 1)(n + 2)$ is not divisible by 5.
We have shown that the value of $n(n + 1)(n + 2)$ is a multiple of 5 when n is: a multiple of 5, or 1 less than a multiple of 5, or 2 less than a multiple of 5.
We have also shown that the value of $n(n + 1)(n + 2)$ is not a multiple of 5 when n is: 3 less than a multiple of 5, or 4 less than a multiple of 5.
Since every positive integer is either a multiple of 5, or 1, 2, 3, or 4 less than a multiple of 5, we have considered the value of $n(n + 1)(n + 2)$ for all positive integers n.
In the first group of 5 positive integers from 1 to 5, there are exactly 3 integers n ($n = 3, 4, 5$) for which $n(n + 1)(n + 2)$ is a multiple of 5.
Similarly, in the second group of 5 positive integers from 6 to 10, there are exactly 3 integers n ($n = 8, 9, 10$) for which $n(n + 1)(n + 2)$ is a multiple of 5.
As was shown above, this pattern continues giving 3 values for n for which $n(n + 1)(n + 2)$ is a multiple of 5 in each successive group of 5 consecutive integers.
When these positive integers, n, are listed in increasing order, we are required to find the 2018^{th} integer in the list.
Since $2018 = 3 \times 672 + 2$, then among the first 672 successive groups of 5 consecutive integers (which is the first $5 \times 672 = 3360$ positive integers), there are exactly $3 \times 672 = 2016$ integers n for which $n(n + 1)(n + 2)$ is a multiple of 5.
The next two integers, 3361 and 3362, do not give values for n for which $n(n + 1)(n + 2)$ is a multiple of 5 (since 3361 is 4 less than a multiple of 5 and 3362 is 3 less than a multiple of 5).
The next two integers, 3363 and 3364, do give values for n for which $n(n+1)(n+2)$ is a multiple of 5.
Therefore, the 2018^{th} integer in the list is 3364.

Answer: (E)

24. Let a, b, c, and d be the four distinct digits that are chosen from the digits 1 to 9.
These four digits can be arranged in 24 different ways to form 24 distinct four-digit numbers.
Consider breaking the solution up into the 3 steps that follow.

Step 1: Determine how many times each of the digits a, b, c, d appears as thousands, hundreds, tens, and units (ones) digits among the 24 four-digit numbers

If one of the 24 four-digit numbers has thousands digit equal to a, then the remaining three digits can be arranged in 6 ways: bcd, bdc, cbd, cdb, dbc, and dcb.
That is, there are exactly 6 four-digit numbers whose thousands digit is a.
If the thousands digit of the four-digit number is b, then the remaining three digits can again be arranged in 6 ways to give 6 different four-digit numbers whose thousands digit is b.
Similarly, there are 6 four-digit numbers whose thousands digit is c and 6 four-digit numbers whose thousands digit is d.
The above reasoning can be used to explain why there are also 6 four-digit numbers whose hundreds digit is a, 6 four-digit numbers whose hundreds digit is b, 6 four-digit numbers whose hundreds digit is c, and 6 four-digit numbers whose hundreds digit is d.
In fact, we can extend this reasoning to conclude that among the 24 four-digit numbers, each of the digits a, b, c, d, appears exactly 6 times as the thousands digit, 6 times as the hundreds digit, 6 times as the tens digit, and 6 times as the units digit.

Step 2: Determine N, the sum of the 24 four-digit numbers

Since each of the digits a, b, c, d appear 6 times as the units digit of the 24 four-digit numbers, then the sum of the units digits of the 24 four-digit numbers is $6a+6b+6c+6d$ or $6\times(a+b+c+d)$.
Similarly, since each of the digits a, b, c, d appear 6 times as the tens digit of the 24 four-digit numbers, then the sum of the tens digits of the 24 four-digit numbers is $10\times 6\times(a+b+c+d)$.
Continuing in this way for the hundreds digits and the thousands digits, we get

$$\begin{aligned} N &= 1000\times 6\times(a+b+c+d)+100\times 6\times(a+b+c+d)+10\times 6\times(a+b+c+d) \\ &\quad + 6\times(a+b+c+d) \\ &= 6000(a+b+c+d)+600(a+b+c+d)+60(a+b+c+d)+6(a+b+c+d) \end{aligned}$$

If we let $s=a+b+c+d$, then $N=6000s+600s+60s+6s=6666s$.

Step 3: Determine the largest sum of the distinct prime factors of $N=6666s$

Writing 6666 as a product of prime numbers, we get $6666=6\times 1111=2\times 3\times 11\times 101$.
So then $N=2\times 3\times 11\times 101\times s$ and thus the sum of the distinct prime factors of N is $2+3+11+101$ added to the prime factors of s which are distinct from $2,3,11$, and 101.
That is, to determine the largest sum of the distinct prime factors of N, we need to find the largest possible sum of the prime factors of s which are not equal to $2,3,11$, and 101.
Since $s=a+b+c+d$ for distinct digits a, b, c, d chosen from the digits 1 to 9, then the largest possible value of s is $9+8+7+6=30$ and the smallest possible value is $1+2+3+4=10$.
If $s=29$ (which occurs when a, b, c, d are equal to $9,8,7,5$ in some order), then $N=2\times 3\times 11\times 101\times 29$ and the sum of the prime factors of N is $2+3+11+101+29=146$ (since 29 is a prime number).
If s is any other integer between 10 and 30 inclusive, the sum of its prime factors is less than 29.
(See if you can convince yourself that all other possible values of s have prime factors whose sum is less than 29. Alternately, you could list the prime factors of each of the integers from 10 to 30 to see that 29 is indeed the largest sum.)
Therefore, the largest sum of the distinct prime factors of N is 146.

Answer: (D)

25. Since the grid has height 2, then there are only two possible lengths for vertical arrows: 1 or 2. Since all arrows in any path have different lengths, then there can be at most 2 vertical arrows in any path.
This means that there cannot be more than 3 horizontal arrows in any path. (If there were 4 or more horizontal arrows then there would have to be 2 consecutive horizontal arrows in the path, which is forbidden by the requirement that two consecutive arrows must be perpendicular.)
This means that any path consists of at most 5 arrows.

Using the restriction that all arrows in any path must have different lengths, we now determine the possible combinations of lengths of vertical arrows and of horizontal arrows to get from A to F.
Once we have determined the possible combinations of vertical and horizontal arrows independently, we then try to combine and arrange them.
First, we look at vertical arrows.
The grid has height 2, and A is 1 unit below F so any combination of vertical arrows in a path must have a net results of 1 unit up.
We use "U" for up and "D" for down.
The possible combinations are:

U1 (up arrow with length 1)
D1, U2 (down arrow with length 1, up arrow with length 2)

Next, we look at horizontal arrows.
The grid has width 12, and A is 9 units to the left of F so any combination of horizontal arrows in a path must have a net result of 9 units right.
We use "R" for right and "L" for left.
Many of these combinations of arrows can be re-arranged in different orders. We will deal with this later.
We proceed by looking at combinations of 1 arrow, then 2 arrows, then 3 arrows.
We note that every combination of vertical arrows includes an arrow with length 1 so we can ignore any horizontal combination that uses an arrow of length 1.
Also, any combination of 3 horizontal arrows must be combined with a combination of 2 horizontal arrows, which have lengths 1 and 2.
Thus, we can ignore any combination of 3 horizontal arrows that includes either or both of an arrow of length 1 and length 2.

a) R9
b) R2, R7
c) R3, R6
d) R4, R5
e) L2, R11
f) L3, R12
g) L3, R4, R8
h) L3, R5, R7
i) L4, R3, R10
j) L4, R5, R8
k) L4, R6, R7
l) L5, R3, R11
m) L5, R4, R10
n) L5, R6, R8
o) L6, R3, R12
p) L6, R4, R11
q) L6, R5, R10
r) L6, R7, R8
s) L7, R4, R12
t) L7, R5, R11
u) L7, R6, R10
v) L8, R5, R12
w) L8, R6, R11
x) L8, R7, R10
y) L9, R6, R12
z) L9, R7, R11
aa) L9, R8, R10
ab) L10, R7, R12
ac) L10, R8, R11
ad) L11, R8, R12

There is only one combination of 1 horizontal arrow.
The combinations of 2 horizontal arrows are listed by including those with two right arrows first (in increasing order of length) and then those with left and right arrows (in increasing order of length).
The combinations of 3 arrows are harder to list completely.
There are no useful combinations that include either 3 right arrows or 2 left arrows, since in either case an arrow of length 1 or 2 would be required.
Here, we have listed combinations of 2 right arrows, then those with "L3" (left arrow of length 3), then those with "L4", and so on.

Now we combine the vertical and horizontal combinations to get the paths.
Each combination of arrow directions and lengths can be drawn to form a path.
Vertical combination U1 can only be combined with horizontal paths a through f, since it cannot be combined with 3 horizontal arrows.

a) There are 2 paths: U1/R9 or R9/U1.

b) There are 2 paths: R2/U1/R7 or R7/U1/R2.

c) Again, there are 2 paths.

d) Again, there are 2 paths.

e) There is 1 path: L2/U1/R11. This is because the arrows must alternate horizontal, vertical, horizontal and we cannot end with a left arrow.

f) Again, there is 1 path.

This is 10 paths so far.
Vertical combination D1, U2 can be combined with horizontal paths of lengths 1, 2 or 3.

a) There is 1 path: D1/R9/U2. This is because we cannot end with a down arrow.

b) Not possible because this would include two arrows of length 2.

c) There are 4 paths: R3/D1/R6/U2, R6/D1/R3/U2, D1/R3/U2/R6, D1/R6/U2/R3. We can interchange R3 and R6 as well as picking whether to start with a vertical or horizontal arrow.

d) Again, there are 4 paths.

e) Not possible because this would include two arrows of length 2.

f) There are 2 paths: L3/D1/R12/U2 and D1/L3/U2/R12.

g) There are 4 paths: L3/D1/R4/U2/R8, R4/D1/L3/U2/R8, L3/D1/R8/U2/R4, R8/D1/L3/U2/R4. Each such combination must start with a horizontal arrow, must end with a right arrow, and must have the down arrow before the up arrow.

h) Again, there are 4 paths.

i) There is 1 path: R3/D1/L4/U2/R10. We cannot begin with R10 or L4 since either would take the path off of the grid, and we must end with an arrow to the right.

j,k) There are 2 paths in each case. For example, with j we have R5/D1/L4/U2/R8 and R8/D1/L4/U2/R5.

l,m) As with i, there is 1 path in each case.

n) As with j, there are 2 paths.

o,p,q) As with i, there is 1 path in each case.

r) As with j, there are 2 paths.

s) to ad) In each of these 12 cases, there is 1 path as with i.

Including the previously counted 10 paths that use U1 only, we have

$$10 + 1 + 4 + 4 + 2 + 4 + 4 + 1 + 2(2) + 2(1) + 2 + 3(1) + 2 + 12(1) = 55$$

paths in total.

Answer: (B)