The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Wednesday, November 21, 2018
(in North America and South America)

Thursday, November 22, 2018
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Do not open this booklet until instructed to do so.
There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name, and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

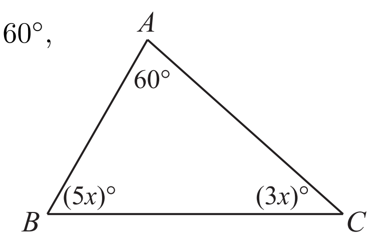
Canadian Intermediate Mathematics Contest

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

PART A

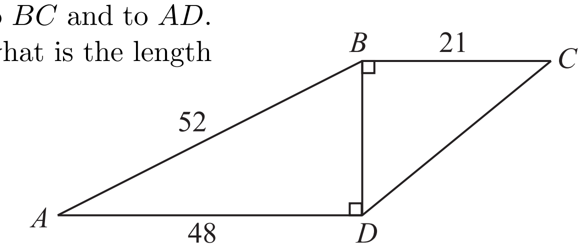
For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. In the diagram, $\triangle ABC$ has angles that measure $60°$, $(5x)°$ and $(3x)°$. What is the value of x?



2. A farm has 500 animals, each of which is one of three types: a goat, a cow, or a chicken. The number of goats is two times the number of cows. If 10% of the animals on the farm are chickens, how many of the animals on the farm are cows?

3. In the diagram, BD is perpendicular to BC and to AD. If $AB = 52$, $BC = 21$, and $AD = 48$, what is the length of DC?



4. The positive integers from 1 to 576 are written in a 24×24 grid so that the first row contains the numbers 1 to 24, the second row contains the numbers 25 to 48, and so on, as shown. An 8×8 square is drawn around 64 of these numbers. (These 64 numbers consist of 8 numbers in each of 8 rows.) The sum of the numbers in the four corners of the 8×8 square is 1646. What is the number in the bottom right corner of this 8×8 square?

1	2	$\cdots$	23	24
25	26	$\cdots$	47	48
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
553	554	$\cdots$	575	576

5. There are five pairs of integers (a, b) with $0 \leq a \leq 10$ and $0 \leq b \leq 10$ for which the points $P(1,1)$, $Q(4,5)$ and $R(a,b)$ form a triangle that has area 6. What are these five pairs of integers (a, b)?

6. There are 20 chairs arranged in a circle. There are n people sitting in n different chairs. These n people stand, move k chairs clockwise, and then sit again. After this happens, exactly the same set of chairs is occupied. (For example, if the 2nd, 4th, 7th, 9th, 12th, 14th, 17th, and 19th chairs are occupied to begin with, then exactly the same set of $n = 8$ chairs is occupied after each person moves $k = 15$ chairs clockwise.) For how many pairs (n, k) with $1 \leq n \leq 20$ and $1 \leq k \leq 20$ is this possible?

PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. The *range* of a list of numbers is the difference between the largest number and the smallest number in the list. For example, the range of the list $1, 5, 1, 6, 3$ is $6 - 1 = 5$.

   (a) Determine the range of the list $7, 13, 4, 9, 6$.

   (b) The list $11, 5, a, 13, 10$ has a range of 12. Determine the two possible values of a.

   (c) The list $6+2x^2, 6+4x^2, 6, 6+5x^2$ has a range of 80. Determine the two possible values of x.

   (d) The list $5x + 3y, 0, x + y, 3x + y$ has a range of 19. If x and y are integers with $x > 0$ and $y > 0$, determine the values of x and y.

2. A bag contains n balls numbered from 1 to n, with $n \geq 2$. There are $n \times (n - 1)$ ways in which Julio can remove one ball from the bag and then remove a second ball. This is because there are n possible choices for the first ball and then $n - 1$ possible choices for the second ball. For example, when a bag contains 6 balls numbered from 1 to 6, 4 of which are black and 2 of which are gold, there are $6 \times 5 = 30$ ways in which he can remove two balls in this way, and $4 \times 3 = 12$ ways in which both balls are black.

   (a) A bag contains 11 balls numbered from 1 to 11, 7 of which are black and 4 of which are gold. Julio removes two balls as described above. What is the probability that both balls are black?

   (b) For some integer $g \geq 2$, a second bag contains 6 black balls and g gold balls; the balls are numbered from 1 to $g + 6$. Julio removes two balls as described above. The probability that both balls are black is $\frac{1}{8}$. Determine the value of g.

   (c) For some integer $x \geq 2$, a third bag contains $2x$ black balls and x gold balls; the balls are numbered from 1 to $3x$. Julio removes two balls as described above. The probability that both balls are black is $\frac{7}{16}$. Determine the value of x.

   (d) For some integer $r \geq 3$, a fourth bag contains 10 black balls, 18 gold balls, and r red balls; the balls are numbered from 1 to $r + 28$. This time, Julio draws three balls one after another. The probability that two of these three balls are black and one of these three balls is gold is at least $\frac{1}{3000}$. What is the largest possible value of r?

3. In Figure 1, $\triangle ABC$ has points D and E on AB and AC, respectively, so that DE is parallel to BC. In this case, $\triangle ABC$ and $\triangle ADE$ are *similar triangles* because their corresponding angles are equal. These triangles thus have the property that $\dfrac{AD}{AB} = \dfrac{AE}{AC} = \dfrac{DE}{BC}$.

In Figure 2, $WXYZ$ is a trapezoid with WX parallel to ZY. Also, points M and N and are on WZ and XY, respectively, with MN parallel to WX and ZY.

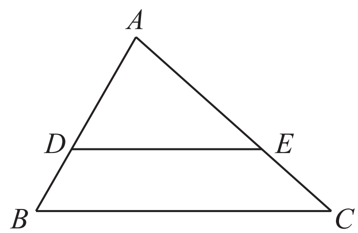


Figure 1

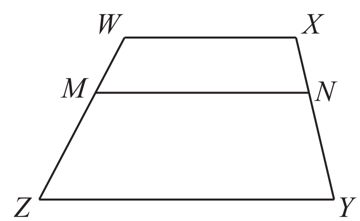


Figure 2

(a) Suppose that, in Figure 1, $DE = 6$, $BC = 10$, $EC = 3$, and $AE = x$. Determine the value of x.

(b) Suppose that, in Figure 2, $\dfrac{WX}{ZY} = \dfrac{3}{4}$ and $\dfrac{WM}{MZ} = \dfrac{XN}{NY} = \dfrac{2}{3}$. Determine the value of $\dfrac{WX}{MN}$.

(c) Suppose that, in Figure 2, $\dfrac{WX}{ZY} = \dfrac{3}{4}$. Suppose also that $\dfrac{MZ}{WM} = \dfrac{NY}{XN}$ and that this ratio is equal to a positive integer. If $WX + MN + ZY = 2541$ and the length of each of WX, MN and ZY is an integer, determine all possible lengths of MN.