The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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2017 Canadian Team Mathematics Contest

Answer Key for Team Problems

Question	Answer
1	40°
2	$\frac{2}{5}$
3	15
4	89
5	192
6	1950 cm^2
7	315°
8	3000π cm^3
9	1584
10	296
11	7
12	20
13	$0, 1$
14	$\frac{13}{4}$
15	$\frac{1}{3}$
16	$\tan\frac{4}{3}$
17	34
18	$\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$
19	$-5, 0, 5$
20	$126^2 = 15\,876$
21	2:00 a.m., 5:00 a.m., 1:00 p.m.
22	2916
23	$\frac{1}{9}$
24	$-\frac{1}{2}, 9 \pm 3\sqrt{10}$
25	$\frac{12}{\sqrt{41}}$

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Answer Key for Individual Problems

Question	Answer
1	4
2	50
3	196 cm
4	$\frac{12}{90}$
5	2 minutes
6	78
7	$\frac{48}{5}$ minutes
8	$6+6\sqrt{2}$
9	4031
10	6595

Answer Key for Relays

Question	Answer
0	$5, 70, 40^\circ$
1	$5, 20, 4$
2	$20, \frac{25}{64}, \frac{64}{13}$
3	$3, 6, 127$

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2017
Canadian Team Mathematics Contest

April 2017

Solutions

Individual Problems

1. If $\dfrac{8}{x} + 6 = 8$, then $\dfrac{8}{x} = 2$ and so $x = 4$.

Answer: 4

2. Since AB and CD are parallel, then $\angle BAF$ and $\angle AFD$ are co-interior angles.
Therefore, $\angle BAF + \angle AFD = 180^\circ$, and so $30^\circ + 3x^\circ = 180^\circ$.
This gives $3x = 150$ or $x = 50$.

Answer: 50

3. Since Ivan and Jackie are each 175 cm tall, then their average height is 175 cm.
We are told that the average height of Ivan, Jackie and Ken together is 4% larger than 175 cm, which equals 1.04×175 cm $= 182$ cm.
Since the average height of 3 people is 182 cm, then the sum of their heights is 3×182 cm or 546 cm.
Since Ivan and Jackie are each 175 cm tall, then Ken's height is $546\text{ cm} - 2 \times 175\text{ cm} = 196$ cm.

Answer: 196 cm

4. There are $99 - 10 + 1 = 90$ positive integers between 10 and 99, inclusive.
If a positive integer is between 10 and 99, the minimum possible sum of its digits is $1 + 0 = 1$ and the maximum possible sum of its digits is $9 + 9 = 18$.
The multiples of 7 in the range 1 to 18 are 7 and 14.
This means that we want to determine the number of positive integers between 10 and 99, inclusive, with sum of digits equal to 7 or 14.
The integers with sum of digits equal to 7 are 16, 25, 34, 43, 52, 61, 70, of which there are 7.
The integers with sum of digits equal to 14 are 59, 68, 77, 86, 95, of which there are 5.
Thus, there are $7 + 5 = 12$ integers in the desired range with sum of digits divisible by 7, and so the probability of choosing one of these integers at random is $\frac{12}{90} = \frac{2}{15}$.

Answer: $\frac{2}{15}$

5. The car takes 10 minutes to travel from the point at which the minivan passes it until it arrives in Betatown.
Since the car drives at 40 km/h and since 10 minutes equals $\frac{1}{6}$ hour, then the car travels 40 km/h $\cdot \frac{1}{6}$ h $= \frac{20}{3}$ km in these 10 minutes.
Thus, the distance between the point where the vehicles pass and Betatown is $\frac{20}{3}$ km.
Since the minivan travels at 50 km/h, it covers this distance in $\dfrac{203\text{ km}}{50\text{ km/h}} = \frac{2}{15}$ h.
Now $\frac{2}{15}$ h $= \frac{8}{60}$ h which equals 8 minutes, and so the minivan arrives in Betatown $10 - 8 = 2$ minutes before the car.

Answer: 2 minutes

6. Since Ruxandra wants to visit 5 countries, then there are $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ orders in which she can visit them with no restrictions.
Of these ways, there are $1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways in which she visits Mongolia first. (Visiting Mongolia first is fixed and so there is 1 choice for the first country; there are still 4 choices for the second country, 3 for the second country, and so on.)
Similarly, there are 24 ways in which she visits Bhutan last.
Both of these totals of 24 include the ways in which she visits Mongolia first and visits Bhutan last. There are 6 such ways, since there is 1 choice for the first country and 1 choice for the

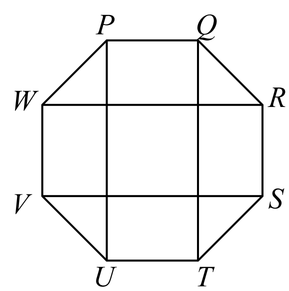
last country, and then 3 choices for the second country, 2 choices for the third country, and 1 for the fourth country.
Therefore, the number of ways in which she either visits Mongolia first or Bhutan last (or both) is $24 + 24 - 6 = 42$.
Therefore, the number of ways with the condition that she does not visit Mongolia first and she does not visit Bhutan last is $120 - 42 = 78$.

Answer: 78

7. Since taps A, B and C can fill the bucket in 16 minutes, 12 minutes and 8 minutes, respectively, then in 1 minute taps A, B and C fill $\frac{1}{16}$, $\frac{1}{12}$ and $\frac{1}{8}$ of the bucket, respectively.
Similarly, in 1 minute, $\frac{1}{6}$ of the bucket drains out through the hole.
Therefore, in 1 minute with the three taps on and the hole open, the fraction of the bucket that fills is $\frac{1}{16} + \frac{1}{12} + \frac{1}{8} - \frac{1}{6} = \frac{3+4+6-8}{48} = \frac{5}{48}$.
Thus, to fill the bucket under these conditions will take $\frac{48}{5}$ minutes.

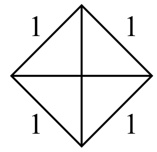
Answer: $\frac{48}{5}$ minutes

8. Let A be the area of a regular octagon with side length 1.
Then the area of a regular octagon with side length 2 equals $2^2A = 4A$.
Thus, the area between the octagons is $4A - A = 3A$.
This means that it is sufficient to find the area of a regular octagon with side length 1.
Label the octagon $PQRSTUVW$. Join PU, QT, WR, VS.



We note that the sum of the angles in an octagon is $6 \cdot 180° = 1080°$ and so in a regular octagon, each interior angle equals $\frac{1}{8}(1080°) = 135°$.
Therefore, $\angle WPQ = \angle PQR = 135°$.
By symmetry, $\angle PWR = \angle QRW$. Looking at quadrilateral $PQRW$, each of these angles equals $\frac{1}{2}(360° - 2 \cdot 135°) = 45°$.
Since $\angle PWR + \angle WPQ = 45° + 135° = 180°$, then PQ is parallel to WR. Using a similar argument, PQ, WR, VS, and UT are all parallel, as are WV, PU, QT, and RS.
Since $\angle PWR = 45°$ and $\angle PWV = 135°$, then $\angle RWV = 90°$, which means that WR is perpendicular to WV.
This means that PQ, WR, VS, and UT are all perpendicular to WV, PU, QT, and RS.
Therefore, the innermost quadrilateral is a square with side length 1. (Note that its sides are parallel and equal in length to PQ, RS, UT, WV, each of which has length 1.) Its area is 1.
The four right-angled triangles with hypotenuses QR, ST, UV, and WP are all isosceles right-angled triangles with hypotenuse of length 1.

These can be pieced together to form a square with side length 1, as shown.



Therefore, their combined area is 1.
The four remaining rectangles are identical. Each has one side of length 1 (one of the sides of the octagon) and one side equal to the shorter side length of an isosceles right-angled triangle with hypotenuse 1, which equals $\frac{1}{\sqrt{2}}$.
Therefore, the combined area of these rectangles is $4 \cdot 1 \cdot \frac{1}{\sqrt{2}} = 2\sqrt{2}$.
Putting these pieces together, we obtain $A = 2 + 2\sqrt{2}$, and so the area between the octagons is $3A = 6 + 6\sqrt{2}$.

ANSWER: $6 + 6\sqrt{2}$

9. Let M be the midpoint of AB.
Let the coordinates of A, B and M be (x_A, y_A), (x_B, y_B) and (x_M, y_M), respectively.
Since M is the midpoint of AB, then $x_M = \dfrac{x_A + x_B}{2}$ and $y_M = \dfrac{y_A + y_B}{2}$.
We want $x_M + y_M = 2017$, which is equivalent to $\dfrac{x_A + x_B}{2} + \dfrac{y_A + y_B}{2} = 2017$.
Thus, $x_A + x_B + y_A + y_B = 4034$.
Since A and B are the points of intersection of the parabolas with equations $y = x^2 - 2x - 3$ and $y = -x^2 + 4x + c$, then x_A and x_B are the solutions of the equation $x^2 - 2x - 3 = -x^2 + 4x + c$ or $2x^2 - 6x - (3 + c) = 0$ and so $x^2 - 3x - \frac{1}{2}(3 + c) = 0$.
The sum of the roots of this quadratic equation is 3 and the product of the roots is $-\frac{1}{2}(3 + c)$.
In other words, $x_A + x_B = 3$ and $x_A x_B = -\frac{1}{2}(3 + c)$.
Since points $A(x_A, y_A)$ and $B(x_B, y_B)$ are on the parabola with equation $y = x^2 - 2x - 3$, then $y_A = x_A^2 - 2x_A - 3$ and $y_B = x_B^2 - 2x_B - 3$.
Therefore, the following equations are equivalent:

$$\begin{aligned}
x_A + x_B + y_A + y_B &= 4034 \\
x_A + x_B + (x_A^2 - 2x_A - 3) + (x_B^2 - 2x_B - 3) &= 4034 \\
x_A^2 + x_B^2 - (x_A + x_B) - 6 &= 4034 \\
(x_A + x_B)^2 - 2x_A x_B - 3 - 6 &= 4034 \\
3^2 - 2\left(-\tfrac{1}{2}(3 + c)\right) &= 4043 \\
3 + c &= 4034
\end{aligned}$$

and so $c = 4031$.

ANSWER: 4031

10. Since the line with equation $b = ax - 4y$ passes through the point $(r, 0)$, then $b = ar - 0$ and so $r = \dfrac{b}{a}$.
Since $0 \leq r \leq 3$, then $0 \leq \dfrac{b}{a} \leq 3$ and so $0 \leq b \leq 3a$. (Since $a > 0$, we can multiply the inequalities by a without switching the direction of the inequalities.)
Since the line with equation $b = ax - 4y$ passes through the point $(s, 4)$, then $b = as - 4(4)$

and so $as = b + 16$ or $s = \dfrac{b+16}{a}$.
Since $2 \le s \le 4$, then $2 \le \dfrac{b+16}{a} \le 4$ and so $2a \le b + 16 \le 4a$ (again, $a > 0$).
Rearranging, we get $2a - 16 \le b \le 4a - 16$.
We now proceed by determining, for each integer a with $1 \le a \le 100$, the number of values of b that satisfy both $0 \le b \le 3a$ and $2a - 16 \le b \le 4a - 16$.
To do this, we need to compare the possible lower and upper bounds in these inequalities.
We note that, to satisfy both pairs of inequalities, we need both $0 \le b$ and $2a - 16 \le b$ as well as both $b \le 3a$ and $b \le 4a - 16$.
Thus, we need to compare 0 with $2a - 16$ and $3a$ with $4a - 16$.
We note that $0 \le 2a - 16$ is equivalent to $16 \le 2a$ or $8 \le a$.
Therefore, when $1 \le a \le 7$, to satisfy both $0 \le b$ and $2a - 16 \le b$, it is sufficient to look at $0 \le b$, and when $8 \le a \le 100$, it is sufficient to look at $2a - 16 \le b$.
We note that $3a \le 4a - 16$ is equivalent to $16 \le a$.
Therefore, when $1 \le a \le 15$, it is sufficient to look at $b \le 4a - 16$ and when $16 \le a \le 100$, it is sufficient to look at $b \le 3a$.
Putting this all together:

- When $1 \le a \le 7$, it is sufficient to count the values of b that satisfy $0 \le b \le 4a - 16$.
  When $a = 1, 2, 3$, $4a - 16$ is negative, so there are no values of b that work.
  When $a = 4$, we obtain $0 \le b \le 0$, and so there is 1 value of b.
  When $a = 5$, we obtain $0 \le b \le 4$, and so there are 5 values of b.
  When $a = 6$, we obtain $0 \le b \le 8$, and so there are 9 values of b.
  When $a = 7$, we obtain $0 \le b \le 12$, and so there are 13 values of b.
  Since $1 + 5 + 9 + 13 = 28$, then there are 28 pairs (a, b) in this case.
- When $8 \le a \le 15$, it is sufficient to count the values of b that satisfy $2a-16 \le b \le 4a-16$.
  For each of these a, there are $(4a - 16) - (2a - 16) + 1 = 2a + 1$ values of b.
  Thus, as a ranges from 8 to 15, there are $17+19+21+23+25+27+29+31$ values of b.
  Since $17+19+21+23+25+27+29+31 = \frac{8}{2}(17+31) = 192$, there are 192 pairs (a, b) in this case.
- When $16 \le a \le 100$, it is sufficient to count the values of b that satisfy $2a - 16 \le b \le 3a$.
  For each of these a, there are $3a - (2a - 16) + 1 = a + 17$ values of b.
  Thus, as a ranges from 16 to 100, there are $33 + 34 + 35 + \cdots + 115 + 116 + 117$ values of b and so this many pairs (a, b).
  Since $33 + 34 + 35 + \cdots + 115 + 116 + 117 = \frac{85}{2}(33 + 117) = 85(75) = 6375$, then there are 6375 pairs (a, b) in this case.

Having considered all cases, we see that there are $28 + 192 + 6375 = 6595$ pairs in total that satisfy the desired conditions.

ANSWER: 6595

Team Problems

1. Since ABC is a straight angle, then $\angle DBA = 180^\circ - \angle DBC = 180^\circ - 130^\circ = 50^\circ$.
Since the angles in a triangle add to 180°, then
$$\angle DAB = 180^\circ - \angle ADB - \angle DBA = 180^\circ - 90^\circ - 50^\circ = 40^\circ$$
(Alternatively, we could note that $\angle DBC$ is an exterior angle for the triangle.
This means that $\angle DBC = \angle DAB + \angle ADB$, which gives $\angle DAB = \angle DBC - \angle ADB = 130^\circ - 90^\circ = 40^\circ$.
Answer: 40°

2. Evaluating,
$$\left[\left(\frac{2017+2017}{2017}\right)^{-1} + \left(\frac{2018}{2018+2018}\right)^{-1}\right]^{-1} = \left[2^{-1} + \left(\frac{1}{2}\right)^{-1}\right]^{-1} = \left[\frac{1}{2}+2\right]^{-1} = \left[\frac{5}{2}\right]^{-1} = \frac{2}{5}$$
Answer: $\frac{2}{5}$

3. The six expressions that Bethany creates are
$$\begin{aligned}
2+0-1\times 7 &= 2+0-7 = -5\\
2+0\times 1-7 &= 2+0-7 = -5\\
2-0+1\times 7 &= 2-0+7 = 9\\
2-0\times 1+7 &= 2-0+7 = 9\\
2\times 0+1-7 &= 0+1-7 = -6\\
2\times 0-1+7 &= 0-1+7 = 6
\end{aligned}$$
Of these, the maximum value is $M = 9$ and the minimum value is $m = -6$.
Thus, $M - m = 9 - (-6) = 15$.
Answer: 15

4. Since $\sqrt{2018} \approx 44.92$, the largest perfect square less than 2018 is $44^2 = 1936$ and the smallest perfect square greater than 2018 is $45^2 = 2025$.
Therefore, $m^2 = 2025$ and $n^2 = 1936$, which gives $m^2 - n^2 = 2025 - 1936 = 89$.
Answer: 89

5. Since $\sqrt{12} + \sqrt{108} = \sqrt{N}$, then $\sqrt{2^2\cdot 3} + \sqrt{6^2\cdot 3} = \sqrt{N}$ or $2\sqrt{3} + 6\sqrt{3} = \sqrt{N}$.
This means that $\sqrt{N} = 8\sqrt{3} = \sqrt{8^2\cdot 3} = \sqrt{192}$ and so $N = 192$.
Answer: 192

6. Since the ratio of the width to the height is $3:2$, we let the width be $3x$ cm and the height be $2x$ cm, for some $x > 0$.
In terms of x, the area of the screen is $(3x \text{ cm})(2x \text{ cm}) = 6x^2 \text{ cm}^2$.
Since the diagonal has length 65 cm, then by the Pythagorean Theorem, we obtain $(3x)^2 + (2x)^2 = 65^2$.
Simplifying, we obtain $9x^2 + 4x^2 = 65^2$ or $13x^2 = 65 \cdot 65$.
Thus, $x^2 = 5 \cdot 65$ and so $6x^2 = 6 \cdot 5 \cdot 65 = 30 \cdot 65 = 1950$.
Therefore, the area of the screen is 1950 cm^2.
Answer: 1950 cm^2

7. Since the three wheels touch and rotate without slipping, then the arc lengths through which each rotates will all be equal.
Since wheel A has radius 35 cm and rotates through an angle of 72°, then the arc length through which it rotates is $\frac{72^\circ}{360^\circ}(2\pi(35\text{ cm})) = 14\pi$ cm. (This equals the fraction of the entire circumference as determined by the fraction of the entire central angle.)
If wheel C rotates through an angle of θ, then we must have $\frac{\theta}{360^\circ}(2\pi(8\text{ cm})) = 14\pi$ cm.
Simplifying, we obtain $\frac{\theta}{360^\circ} = \frac{14}{16}$ and so $\theta = 360^\circ \cdot \frac{7}{8} = 315^\circ$.
Therefore, wheel C rotates through an angle of 315°.

Answer: 315°

8. The volume of a cylinder with radius r and height h is $\pi r^2 h$.
The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.
The given cylinder has radius 10 cm and height 70 cm and so has volume $\pi(10\text{ cm})^2(70\text{ cm}) = 7000\pi\text{ cm}^3$.
Since the radius of the spheres and the radius of the cylinder are equal, then we can view each of the spheres as having a "height" of $2 \cdot 10$ cm $= 20$ cm. (In other words, the spheres can only stack perfectly on top of each other.)
Since the height of the cylinder is 70 cm and the height of each sphere is 20 cm, then a maximum of 3 spheres will fit inside the closed cylinder.
Therefore, the volume of the cylinder not taken up by the spheres is $7000\pi\text{ cm}^3 - 3\cdot\frac{4}{3}\pi(10\text{ cm})^3$ or $7000\pi\text{ cm}^3 - 4000\pi\text{ cm}^3$ or $3000\pi\text{ cm}^3$.

Answer: $3000\pi\text{ cm}^3$

9. Removing the initial 0, the remaining 99 terms can be written in groups of the form $(3k-2)+(3k-1)-3k$ for each k from 1 to 33.
The expression $(3k-2)+(3k-1)-3k$ simplifies to $3k-3$.
Therefore, the given sum equals

$$0 + 3 + 6 + \cdots + 93 + 96$$

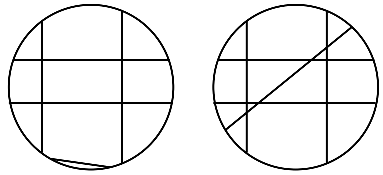
Since k ran from 1 to 33, then this sum includes 33 terms and so equals

$$\frac{33}{2}(0+96) = 33(48) = 33(50) - 33(2) = 1650 - 66 = 1584$$

Answer: 1584

10. When the 5th chord is added, it is possible that it creates only 1 new region, which means that $m = 9 + 1 = 10$.
Since there are already 4 chords, then the maximum possible number of chords that the 5th chord intersects is 4.
If the 5th chords intersects 4 chords, then it passes through 5 regions (one before the first intersection and one after each intersection) and it splits each of the 5 regions into 2 regions, which creates 5 new regions.
Since the 5th chord cannot intersect more than 4 chords, it cannot pass through more than 5 regions.

Diagrams that show each of these cases are shown below:



Therefore, $M = 9 + 5 = 14$ and so $M^2 + m^2 = 14^2 + 10^2 = 296$.

ANSWER: 296

11. The product of the roots of the quadratic equation $ax^2 + bx + c = 0$ is $\frac{c}{a}$ and the sum of the roots is $-\frac{b}{a}$.
Since the product of the roots of $2x^2 + px - p + 4 = 0$ is 9, then $\frac{-p+4}{2} = 9$ and so $-p + 4 = 18$, which gives $p = -14$.
Therefore, the quadratic equation is $2x^2 - 14x + 18 = 0$ and the sum of its roots is $-\frac{(-14)}{2} = 7$.

ANSWER: 7

12. The six pairs are $a + b, a + c, a + d, b + c, b + d, c + d$.
The sum of the two smallest numbers will be the smallest sum. Thus, $a + b = 6$.
The second smallest sum will be $a + c$ since $a < b < c < d$ gives $a + b < a + c$ and $a + c < a + d$ and $a + c < b + c$ and $a + c < b + d$ and $a + c < c + d$. Thus, $a + c = 8$.
Using a similar argument, the largest sum must be $c + d$ and the second largest sum must be $b + d$.
Therefore, the third and fourth sums (as listed in increasing order) are $a + d = 12$ and $b + c = 21$ or $b + c = 12$ and $a + d = 21$.
If $b + c = 21$, then we would have $(a + b) + (a + c) + (b + c) = 6 + 8 + 21$ and so $2a + 2b + 2c = 35$. This cannot be the case since a, b, c are integers, which makes the left side even and the right side odd.
Therefore, $b + c = 12$ and $a + d = 21$.
This gives $(a + b) + (a + c) + (b + c) = 6 + 8 + 12$ or $2a + 2b + 2c = 26$ or $a + b + c = 13$.
Since $b + c = 12$, then $a = (a + b + c) - (b + c) = 13 - 12 = 1$.
Since $a + d = 21$, then $d = 21 - a = 20$.
(We can check that $a = 1$, $b = 5$, $c = 7$, $d = 20$ fit with the given information.)

ANSWER: 20

13. Starting with the given equation, we obtain the following equivalent equations:

$$\begin{aligned} 16^x - \tfrac{5}{2}(2^{2x+1}) + 4 &= 0 \\ (4^2)^x - \tfrac{5}{2}(2 \cdot (2^2)^x) + 4 &= 0 \\ (4^x)^2 - 5(4^x) + 4 &= 0 \\ (4^x - 4)(4^x - 1) &= 0 \end{aligned}$$

Therefore, $4^x = 4$ (which gives $x = 1$) or $4^x = 1$ (which gives $x = 0$).
Thus, the solutions are $x = 0, 1$.

ANSWER: $0, 1$

14. Since $f(k) = 4$ and $f(f(k)) = 7$, then $f(4) = 7$.
Since $f(f(k)) = 7$ and $f(f(f(k))) = 19$, then $f(7) = 19$.
Suppose that $f(x) = ax + b$ for some real numbers a and b.
Since $f(4) = 7$, then $7 = 4a + b$.
Since $f(7) = 19$, then $19 = 7a + b$.
Subtracting, we obtain $(7a + b) - (4a + b) = 19 - 7$ and so $3a = 12$ or $a = 4$, which gives $b = 7 - 4a = 7 - 16 = -9$.
Therefore, $f(x) = 4x - 9$ for all x.
Since $f(k) = 4$, then $4 = 4k - 9$ and so $4k = 13$ or $k = \frac{13}{4}$.

ANSWER: $\frac{13}{4}$

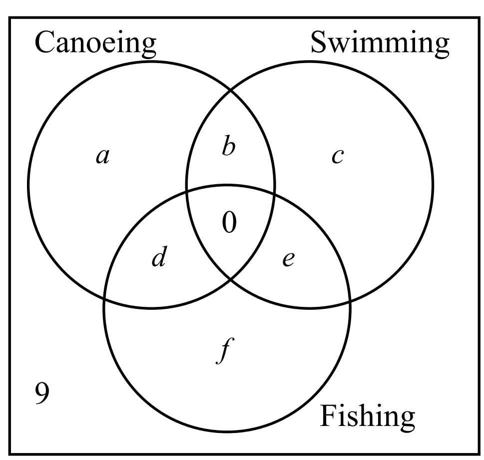
15. Each of the 27 smaller triangular prisms has 3 faces that are squares with side length 1 and 2 faces that are equilateral triangles with side length 1.
Combined, these prisms have $27 \cdot 3 = 81$ square faces and $27 \cdot 2 = 54$ triangular faces.
Each of the 3 square faces of the larger triangular prism is made up of 9 of the smaller square faces, which means that $3 \cdot 9 = 27$ of these smaller square faces are painted.
Each of the 2 triangular faces of the larger triangular prism is made up of 9 of the smaller square faces, which means that $2 \cdot 9 = 18$ of these smaller triangular faces are painted.
Therefore, 27 of the 81 smaller square faces are painted and 18 of the 54 smaller triangular faces are painted. In other words, $\frac{1}{3}$ of each type of face is painted, so $\frac{1}{3}$ of the total surface area of the smaller prisms is painted.

ANSWER: $\frac{1}{3}$

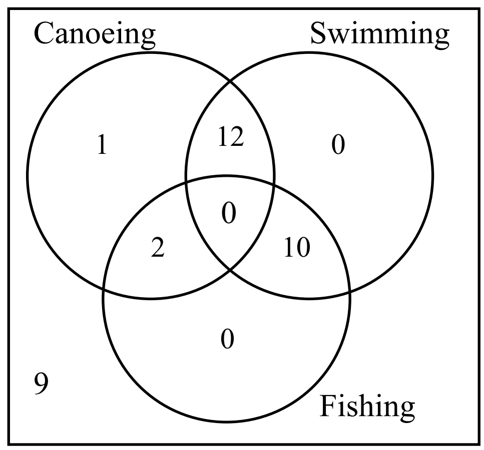
16. Let θ be the angle (in radians) between the line with equation $y = kx$ and the x-axis.
Then $k = \tan\theta$, since the tangent function can be viewed as measuring slope.
The shaded area is equal to the area of a sector with central angle θ of the larger circle with radius 2 minus the area of a sector with central angle θ of the smaller circle with radius 1.
Therefore, $\dfrac{\theta}{2\pi}\pi(2^2) - \dfrac{\theta}{2\pi}\pi(1^2) = 2$.
Multiplying both sides by 2 and dividing out the factor of π in numerators and denominators, we obtain $4\theta - \theta = 4$ and so $3\theta = 4$ or $\theta = \frac{4}{3}$.
Thus, $k = \tan\frac{4}{3}$.

ANSWER: $\tan\frac{4}{3}$

17. We start by creating a Venn diagram:



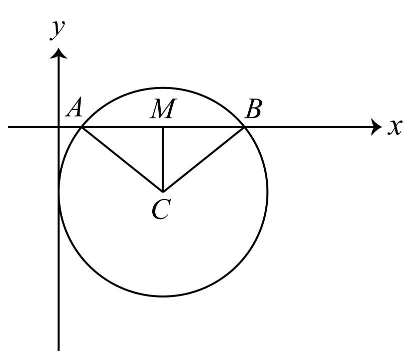
We want to determine the minimum possible value of $a + b + c + d + e + f + 9$.
From the given information $a + b + d = 15$ and $b + c + e = 22$ and $d + e + f = 12$.
Adding these equations, we obtain $a+2b+c+2e+f+2d = 49$ or $a+b+c+d+e+f+(b+d+e) = 49$.
Thus, $a + b + c + d + e + f = 49 - (b + d + e)$ and so we want to minimize $49 - (b + d + e)$, which means that we want to maximize $b + d + e$.
Since a, b, c, d, e, f are non-negative integers, then $a + b + d = 15$ means $b + d \leq 15$. Similarly, $b + e \leq 22$ and $d + e \leq 12$.
Adding these inequalities, we obtain $2b + 2d + 2e \leq 49$ and so $b + d + e \leq \frac{49}{2}$.
Since b, d, e are integers, then in fact $b + d + e \leq 24$.
It is possible to make $b + d + e = 24$ by setting $a = 1$, $c = 0$ and $f = 0$. This gives $b + d = 14$ and $b + e = 22$ and $d + e = 12$, and so $b = 12$ and $d = 2$ and $e = 10$.
We can see this in the completed Venn diagram here:



Therefore, the maximum possible value of $b+d+e$ is 24 and so the minimum possible number of campers is $58 - 24 = 34$.

Answer: 34

18. Let C be the centre of the circle. Note that C is in the fourth quadrant.
Let A and B be the points where the circle intersects the x-axis and let M be the midpoint of AB.



We want to find the area of the region inside the circle and above AB.
We calculate this area by finding the area of sector ACB and subtracting the area of $\triangle ACB$.
Join C to A, B and M.
Since the radius of the circle is 1, then $CA = CB = 1$.
Since M is the midpoint of AB and C is the centre, then CM is perpendicular to AB, which

means that CM is vertical. Since the coordinates of C are $\left(1, -\frac{\sqrt{3}}{2}\right)$, then $CM = \frac{\sqrt{3}}{2}$.
Since $AC : CM = 1 : \frac{\sqrt{3}}{2}$, then $\triangle CAM$ is a $30°$-$60°$-$90°$ triangle, as is $\triangle BCM$.
Therefore, $\angle ACB = 60°$, and thus the area of sector ACB equals $\frac{60°}{360°}\pi(1^2) = \frac{1}{6}\pi$.
Also, since $\triangle CAM$ is $30°$-$60°$-$90°$, then $AM = \frac{1}{2}$. Similarly, $BM = \frac{1}{2}$.
Therefore, the area of $\triangle ACB$ is $\frac{1}{2}AB \cdot CM = \frac{1}{2}(1)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$.
Finally, this means that the area inside the circle and inside the first quadrant is $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$.

ANSWER: $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$

19. When $f(x)$ is divided by $x-4$, the remainder is 102.
This means that $f(x) - 102$ has a factor of $x-4$.
Similarly, $f(x) - 102$ is divisible by each of $x-3$, $x+3$ and $x+4$.
Since $f(x)$ is a quartic polynomial, then $f(x) - 102$ is a quartic polynomial and so can have at most four distinct linear factors.
Since the leading coefficient of $f(x)$ is 1, then the leading coefficient of $f(x) - 102$ is also 1.
This means that $f(x) - 102 = (x-4)(x-3)(x+3)(x+4)$.
Thus, $f(x) = (x-4)(x-3)(x+3)(x+4) + 102$.
Therefore, the following equations are equivalent:

$$\begin{aligned} f(x) &= 246 \\ (x-4)(x-3)(x+3)(x+4) + 102 &= 246 \\ (x-4)(x+4)(x-3)(x+3) &= 144 \\ (x^2-16)(x^2-9) &= 144 \\ x^4 - 25x^2 + 144 &= 144 \\ x^4 - 25x^2 &= 0 \\ x^2(x^2-25) &= 0 \\ x^2(x-5)(x+5) &= 0 \end{aligned}$$

Thus, the values of x for which $f(x) = 246$ are $x = 0, 5, -5$.

ANSWER: $0, 5, -5$

20. Since $-1 \leq \cos\theta \leq 1$ for all angles θ, then $0 \leq \cos^6\theta \leq 1$ for all angles θ.
Also, $\sin^6\alpha \geq 0$ for all angles α.
Therefore, to have $\cos^6\theta - \sin^6\alpha = 1$ for some angles θ and α, then we must have $\cos^6\theta = 1$ and $\sin^6\alpha = 0$.
From the given equation, this means that $\cos^6(1000x) = 1$ and $\sin^6(1000y) = 0$.
Therefore, $\cos(1000x) = \pm 1$ and $\sin(1000y) = 0$.
Since $0 \leq x \leq \frac{\pi}{8}$, then $0 \leq 1000x \leq 125\pi$.
Now $\cos\theta = \pm 1$ exactly when θ is a multiple of π.
Therefore, the possible values of $1000x$ are $0, \pi, 2\pi, \ldots, 124\pi, 125\pi$.
There are 126 such values.
Since $0 \leq y \leq \frac{\pi}{8}$, then $0 \leq 1000y \leq 125\pi$.
Now $\sin\alpha = 0$ exactly when α is a multiple of π.
Therefore, the possible values of $1000y$ are $0, \pi, 2\pi, \ldots, 124\pi, 125\pi$.
There are 126 such values.
Since there are 126 values of x and 126 independent values of y, then there are $126^2 = 15\,876$ pairs (x, y) that satisfy the original equation and restrictions.

ANSWER: 126^2

21. From the given information, Serge writes down the times that are $1+2+3+\cdots+(n-1)+n$ minutes after midnight for each successive value of n from 1 up until the total is large enough to exceed 24 hours in minutes, which equals $24 \cdot 60 = 1440$ minutes.
Now $1+2+3+\cdots+(n-1)+n = \frac{1}{2}n(n+1)$ for each positive integer n.
To find times that are "on the hour", we find the values of n for which $\frac{1}{2}n(n+1)$ is a multiple of 60 (that is, the time is a multiple of 60 minutes after midnight).
Since we want $\frac{1}{2}n(n+1) \leq 1440$, then $n(n+1) \leq 2880$.
Since $52 \cdot 53 = 2756$ and $53 \cdot 54 = 2862$ and $n(n+1)$ increases as n increases, then we only need to check values of n up to and including $n = 52$.
For $\frac{1}{2}n(n+1)$ to be a multiple of 60, we need $\frac{1}{2}n(n+1) = 60k$ for some integer k, or equivalently $n(n+1) = 120k$ for some integer k.
Note that n and $n+1$ are either even and odd, or odd and even, respectively.
Note also that $120k$ is divisible by 8.
Thus, one of n or $n+1$ is divisible by 8.
Since the maximum possible value of n is 52, we can check the possibilities by hand:

n	$n(n+1)$	Divisible by 120?	$\frac{1}{2}n(n+1)$
7	56	No	
8	72	No	
15	240	Yes	120
16	272	No	
23	552	No	
24	600	Yes	300
31	992	No	
32	1056	No	
39	1560	Yes	780
40	1640	No	
47	2256	No	
48	2352	No	

(Note that we could have ruled out the cases $n = 7, 8, 16, 23, 31, 32, 47, 48$ by noting that in these cases neither n nor $n+1$ is divisible by 5 and so $n(n+1)$ cannot be a multiple of 120.)
Therefore, the times that Serge writes down are those that are 120, 300 and 780 minutes after midnight, which are 2, 5 and 13 hours, respectively, after midnight.
That is, Serge writes down 2:00 a.m., 5:00 a.m., and 1:00 p.m.

ANSWER: 2:00 a.m., 5:00 a.m., 1:00 p.m.

22. Figure 0 consists of 1 square with side length 18. Thus, $A_0 = 18^2$.
Figure 1 consists of Figure 0 plus the addition of 2 squares with side length $\frac{2}{3} \cdot 18$.
Thus, $A_1 = 18^2 + 2\left(\frac{2}{3} \cdot 18\right)^2$.
Figure 2 consists of Figure 1 plus the addition of 4 squares with side length $\frac{2}{3} \cdot \frac{2}{3} \cdot 18$.
Thus, $A_2 = 18^2 + 2\left(\frac{2}{3} \cdot 18\right)^2 + 4\left(\frac{2}{3} \cdot \frac{2}{3} \cdot 18\right)^2$.
In general, since twice as many squares are added at each step as at the step before, then 2^n squares are added to Figure $n-1$ to make Figure n.
Also, since the side length of the squares added at each step is $\frac{2}{3}$ of the side length of the squares added at the previous step, then the squares added for Figure n have side length $18\left(\frac{2}{3}\right)^n$.

Therefore,

$$\begin{aligned} A_n &= 18^2 + 2\left(\frac{2}{3}\cdot 18\right)^2 + 4\left(\frac{2}{3}\cdot\frac{2}{3}\cdot 18\right)^2 + \cdots + 2^n\left(18\left(\frac{2}{3}\right)^n\right)^2 \\ &= 18^2 + 2^1\cdot 18^2\left(\frac{2}{3}\right)^2 + 2^2\cdot 18^2\left(\frac{2}{3}\right)^4 + \cdots + 2^n\cdot 18^2\left(\frac{2}{3}\right)^{2n} \end{aligned}$$

The right side is a geometric series with $n+1$ terms, first term $a = 18^2$, and common ratio $r = 2\left(\frac{2}{3}\right)^2 = \frac{8}{9}$.
Therefore,

$$A_n = \frac{18^2\left(1-\left(\frac{8}{9}\right)^{n+1}\right)}{1-\frac{8}{9}} = 9\cdot 18^2\left(1-\left(\tfrac{8}{9}\right)^{n+1}\right) = 2916\left(1-\left(\tfrac{8}{9}\right)^{n+1}\right)$$

Since $\left(\frac{8}{9}\right)^{n+1} > 0$ for all integers $n \geq 0$, then $1-\left(\frac{8}{9}\right)^{n+1} < 1$ for all integers $n \geq 0$.
Therefore, $A_n < 2916\cdot 1 = 2916$ for all integers $n \geq 0$.
But, as n approaches infinity, $\left(\frac{8}{9}\right)^{n+1}$ approaches 0, which means that $1-\left(\frac{8}{9}\right)^{n+1}$ approaches 1, which means that A_n approaches 2916.
In other words, while A_n can never equal 2916, it will eventually be infinitesimally close to 2916. We can check that if $n \geq 68$, then $A_n > 2915$.
Therefore, the smallest positive integers M with the property that $A_n < M$ for all integers $n \geq 0$ is $M = 2916$.

ANSWER: 2916

23. Brad's answer to each question is either right or wrong, and so there are 2^8 possible combinations of right and wrong answers for the remaining 8 questions.

Since Brad has already answered exactly 1 question correctly, then $\binom{8}{4}$ of these combinations will include exactly 5 correct answers. (We choose 4 of the remaining 8 questions to be answered correctly.)

We show that each of these $\binom{8}{4}$ combinations has an equal probability, p.

This will mean that the probability that Brad has exactly 5 correct answers is $\binom{8}{4}p$.

Consider a specific combination that includes exactly 5 correct answers.
For each question from the 3rd to the 10th, the probability that his answer is correct equals the ratio of the number of problems that he has already answered correctly to the total number of problems that he has already answered.
Since the probability that his answer is wrong equals 1 minus the probability that his answer is correct, then the probability that his answer is wrong will equal the number of problems that he has already answered *incorrectly* to the total number of problems that he has already answered. (This is because the total number of problems answered equals the number correct plus the number incorrect.)
Therefore, for each problem from the 3rd to 10th, the associated probability is a fraction with denominator from 2 to 9, respectively (the number of problems already answered).
Consider the positions in this combination in which he gives his 2nd, 3rd, 4th, and 5th correct answers. In these cases, he has previously answered 1, 2, 3, and 4 problems correctly, and so the numerators of the corresponding probability fractions will be 1, 2, 3, and 4.

Similarly, in the positions where his answer is wrong, the numerators will be 1, 2, 3, and 4. Therefore, $p = \dfrac{(1\cdot 2\cdot 3\cdot 4)(1\cdot 2\cdot 3\cdot 4)}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9}$ and so the probability that Brad answers exactly 5 out of the 10 problems correctly is

$$\binom{8}{4}\frac{(1\cdot 2\cdot 3\cdot 4)(1\cdot 2\cdot 3\cdot 4)}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9} = \frac{8!}{4!4!}\cdot\frac{(1\cdot 2\cdot 3\cdot 4)(1\cdot 2\cdot 3\cdot 4)}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9} = \frac{8!}{4!4!}\cdot\frac{4!4!}{9!} = \frac{8!}{9!} = \frac{1}{9}$$

Answer: $\dfrac{1}{9}$

24. We proceed by (very carefully!) calculating the coordinates of points A, B and C, and then equating AB and AC to solve for t.
Point A is the point of intersection of the lines with equations $x + y = 3$ and $2x - y = 0$.
Since $x + y = 3$ and $y = 2x$, then $x + 2x = 3$ and so $x = 1$ which gives $y = 2x = 2$.
Therefore, A has coordinates $(1, 2)$.
Point B is the point of intersection of the lines with equations $x + y = 3$ and $3x - ty = 4$.
Adding t times the first equation to the second, we obtain $tx + 3x = 3t + 4$ and so $x = \dfrac{3t+4}{t+3}$, with the restriction that $t \neq -3$.
Since $x + y = 3$, then $y = 3 - \dfrac{3t+4}{t+3} = \dfrac{3t+9}{t+3} - \dfrac{3t+4}{t+3} = \dfrac{5}{t+3}$.
Therefore, B has coordinates $\left(\dfrac{3t+4}{t+3}, \dfrac{5}{t+3}\right)$.
Point C is the point of intersection of the lines with equations $2x - y = 0$ and $3x - ty = 4$.
Subtracting t times the first equation from the second, we obtain $3x - 2tx = 4 - 0$ and so $x = \dfrac{4}{3-2t}$ with the restriction that $t \neq \frac{3}{2}$.
Since $y = 2x$, then $y = \dfrac{8}{3-2t}$.
Therefore, C has coordinates $\left(\dfrac{4}{3-2t}, \dfrac{8}{3-2t}\right)$.
We assume that $t \neq -3, \frac{3}{2}$.
Since AB and AC are non-negative lengths, then the following equations are equivalent:

$$\begin{aligned}
AB &= AC \\
AB^2 &= AC^2 \\
\left(\frac{3t+4}{t+3} - 1\right)^2 + \left(\frac{5}{t+3} - 2\right)^2 &= \left(\frac{4}{3-2t} - 1\right)^2 + \left(\frac{8}{3-2t} - 2\right)^2 \\
\left(\frac{(3t+4)-(t+3)}{t+3}\right)^2 + \left(\frac{5-(2t+6)}{t+3}\right)^2 &= \left(\frac{4-(3-2t)}{3-2t}\right)^2 + \left(\frac{8-(6-4t)}{3-2t}\right)^2 \\
\left(\frac{2t+1}{t+3}\right)^2 + \left(\frac{-2t-1}{t+3}\right)^2 &= \left(\frac{2t+1}{3-2t}\right)^2 + \left(\frac{4t+2}{3-2t}\right)^2 \\
(2t+1)^2\left[\left(\frac{1}{t+3}\right)^2 + \left(\frac{-1}{t+3}\right)^2\right] &= (2t+1)^2\left[\left(\frac{1}{3-2t}\right)^2 + \left(\frac{2}{3-2t}\right)^2\right]
\end{aligned}$$

$$\begin{aligned}
(2t+1)^2\left[\frac{2}{(t+3)^2}-\frac{5}{(3-2t)^2}\right] &= 0\\
(2t+1)^2\left[\frac{2(3-2t)^2-5(t+3)^2}{(t+3)^2(3-2t)^2}\right] &= 0\\
(2t+1)^2\left[\frac{2(4t^2-12t+9)-5(t^2+6t+9)}{(t+3)^2(3-2t)^2}\right] &= 0\\
(2t+1)^2\left[\frac{3t^2-54t-27}{(t+3)^2(3-2t)^2}\right] &= 0\\
3(2t+1)^2\left[\frac{t^2-18t-9}{(t+3)^2(3-2t)^2}\right] &= 0
\end{aligned}$$

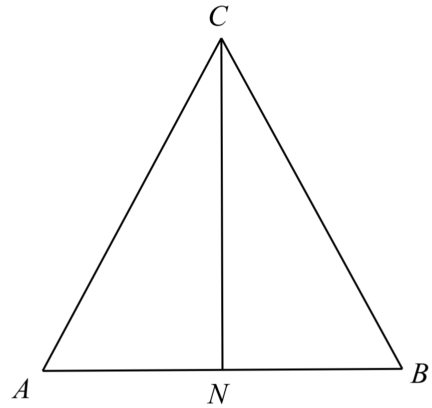
Therefore, the values of t for which $AB = AC$ are $t = -\frac{1}{2}$ and the two values of t for which $t^2 - 18t - 9 = 0$, which are $t = \dfrac{18 \pm \sqrt{18^2 - 4(1)(-9)}}{2} = \dfrac{18 \pm \sqrt{360}}{2} = 9 \pm 3\sqrt{10}$.

ANSWER: $-\frac{1}{2}, 9 \pm 3\sqrt{10}$

25. Since $\triangle ACB$ is isosceles with $AC = BC$ and CD is a median, then CD is perpendicular to AB.
Since CD is perpendicular to AB, then when $\triangle ABC$ is folded along CD, CD will be perpendicular to the plane of $\triangle ADB$.
Therefore, the volume of the tent can be calculated as $\frac{1}{3}$ times the area of $\triangle ADB$ times CD.
Since CD is fixed in length, then the volume is maximized when the area of $\triangle ADB$ is maximized.
Since $AD = DB$ are fixed in length, the area of $\triangle ADB$ is maximized when $\angle ADB = 90^\circ$. (One way to see this is to note that we can calculate the area of $\triangle ADB$ as $\frac{1}{2}(AD)(DB)\sin(\angle ADB)$, which is maximized when $\sin(\angle ADB) = 1$.)
Since $AB = 6$ m, then $AD = DB = 3$ m.
Since $BD = 3$ m and $BC = 5$ m, then $CD = 4$ m. ($\triangle BDC$ is a 3-4-5 triangle.)
Thus, the maximum volume of the tent is

$$\tfrac{1}{3}(\tfrac{1}{2}(AD)(DB))(DC) = \tfrac{1}{6}(AD)(DB)(DC) = \tfrac{1}{6}(3\text{ m})(3\text{ m})(4\text{ m}) = 6\text{ m}^3$$

To find the height, h, of D above the base $\triangle ABC$, we note that the volume equals $\frac{1}{3}$ times the area of $\triangle ABC$ times the height, h.
Since we know that the volume is 6 m^3, then if we calculate the area of $\triangle ABC$, we can solve for h.
Since $AD = DB = 3$ m and $\angle ADB = 90^\circ$, then $AB = 3\sqrt{2}$ m in the maximum volume configuration.
Therefore, $\triangle ABC$ has $AB = 3\sqrt{2}$ m and $AC = BC = 5$ m.
Let N be the midpoint of AB.



Since $AC = BC$, then CN is perpendicular to AB.
Since $BC = 5$ m and $NB = \frac{3\sqrt{2}}{2}$ m, then by the Pythagorean Theorem,

$$CN^2 = BC^2 - NB^2 = (5\text{ m})^2 - \left(\frac{3\sqrt{2}}{2}\text{ m}\right)^2 = 25\text{ m}^2 - \frac{18}{4}\text{ m}^2 = 25\text{ m}^2 - \frac{9}{2}\text{ m}^2 = \frac{41}{2}\text{ m}^2$$

and so $CN = \sqrt{\frac{41}{2}}$ m.
Thus, the area of $\triangle ABC$ is $\frac{1}{2}(AB)(CN) = \frac{1}{2}\left(3\sqrt{2}\text{ m}\right)\left(\frac{\sqrt{41}}{\sqrt{2}}\text{ m}\right) = \frac{3\sqrt{41}}{2}\text{ m}^2$.
Finally, we have $\frac{1}{3}\left(\frac{3\sqrt{41}}{2}\text{ m}^2\right)h = 6\text{ m}^3$ and so $h = \frac{12}{\sqrt{41}}$ m.

ANSWER: $\frac{12}{\sqrt{41}}$ m

Relay Problems

(Note: Where possible, the solutions to parts (b) and (c) of each Relay are written as if the value of t is not initially known, and then t is substituted at the end.)

0. (a) Evaluating, $\dfrac{9+2\times 3}{3} = \dfrac{9+6}{3} = \dfrac{15}{3} = 5$.

   (b) The area of a triangle with base $2t$ and height $3t-1$ is $\frac{1}{2}(2t)(3t-1)$ or $t(3t-1)$.
   Since the answer to (a) is 5, then $t=5$, and so $t(3t-1) = 5(14) = 70$.

   (c) Since $AB = BC$, then $\angle BCA = \angle BAC = t^\circ$.
   Therefore, $\angle ABC = 180^\circ - \angle BCA - \angle BAC = 180^\circ - 2t^\circ$.
   Since the answer to (b) is 70, then $t = 70$, and so

$$\angle ABC = 180^\circ - 2\cdot 70^\circ = 180^\circ - 140^\circ = 40^\circ$$

Answer: $5, 70, 40^\circ$

1. (a) Since $w^2 - 5w = 0$, then $w(w-5) = 0$ and so $w = 0$ or $w = 5$.
   Since w is positive, then $w = 5$.

   (b) The area of the shaded region equals the difference of the areas of the two squares, or $(2t-4)^2 - 4^2$.
   Simplifying, we obtain $(2t-4)^2 - 4^2 = 4t^2 - 16t + 16 - 16 = 4t^2 - 16t$.
   Since the answer to (a) is 5, then $t = 5$, and so $4t^2 - 16t = 4(5^2) - 16(5) = 100 - 80 = 20$.

   (c) The positive integer with digits $xy0$ equals $10\cdot xy$.
   Therefore, such an integer is divisible by 11 exactly when xy is divisible by 11.
   xy is divisible by 11 exactly when $x = y$.
   The positive integers $xy0$ that are divisible by 11 are

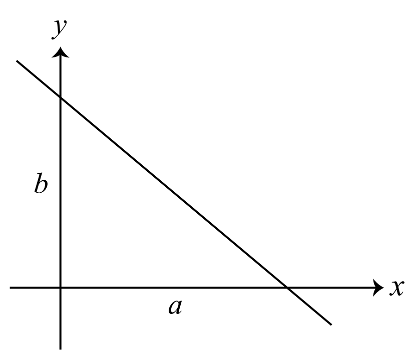
$$110, 220, 330, 440, 550, 660, 770, 880, 990$$

   Since the answer to (b) is 20, then $t = 20$ and so we want to find the numbers in this list that are divisible by 20.
   These are 220, 440, 660, 880. There are 4 such integers.

Answer: $5, 20, 4$

2. (a) We note that $300^8 = 3^8 \cdot 100^8 = 3^8 \cdot (10^2)^8 = 6561\cdot 10^{16}$.
   Multiplying 6561 by 10^{16} is equivalent to appending 16 zeroes to the right end of 6561, creating an integer with 20 digits.

   (b) Let a be the x-intercept of the line with equation $\sqrt{k}x + 4y = 10$ and let b be the y-intercept of this line.



Then the area of the triangle formed by the line and the axes is $\dfrac{ab}{2}$.

Since a is the x-intercept, then $\sqrt{k}a + 4(0) = 10$ or $a = \dfrac{10}{\sqrt{k}}$.

Since b is the y-intercept, then $\sqrt{k}0 + 4(b) = 10$ or $b = \dfrac{10}{4}$.

Since we are given that the area is t, then $t = \dfrac{ab}{2} = \dfrac{10 \cdot 10}{2 \cdot \sqrt{k} \cdot 4}$.

Thus, $\sqrt{k} = \dfrac{100}{8t} = \dfrac{25}{2t}$.

Since the answer to (a) is 20, then $t = 20$ and so $\sqrt{k} = \dfrac{25}{2(20)} = \dfrac{5}{8}$ and so $k = \dfrac{25}{64}$.

(c) Let the height of the spruce tree be s m.
From the given information, the height of the pine tree is $(s-4)$ m, and so the height of the maple tree is $(s-4)$ m $+ 1$ m $= (s-3)$ m.

Thus, the ratio of the height of the maple tree to the height of the spruce tree is $\dfrac{s-3}{s} = t$.

Simplifying, we obtain $1 - \dfrac{3}{s} = t$ and so $1 - t = \dfrac{3}{s}$ which gives $s = \dfrac{3}{1-t}$.

Since the answer to (b) is $\frac{25}{64}$, then $s = \dfrac{3}{1 - \frac{25}{64}} = \dfrac{3}{39/64} = \dfrac{64}{13}$.

Answer: $20, \frac{25}{64}, \frac{64}{13}$

3. (a) Evaluating, $x = \sqrt{20 - 17 - 2 \times 0 - 1 + 7} = \sqrt{20 - 17 - 0 - 1 + 7} = \sqrt{9} = 3$.

(b) Since the graph of $y = 2\sqrt{2t}\sqrt{x} - 2t$ passes through the point (a, a), then $a = 2\sqrt{2t}\sqrt{a} - 2t$.
Rearranging, we obtain $a - 2\sqrt{2t}\sqrt{a} + 2t = 0$. We re-write as $(\sqrt{a})^2 - 2\sqrt{a}\sqrt{2t} + (\sqrt{2t})^2 = 0$ or $(\sqrt{a} - \sqrt{2t})^2 = 0$.
Therefore, $\sqrt{a} = \sqrt{2t}$ or $a = 2t$.
Since the answer to (a) is 3, then $t = 3$ and so $a = 6$.

(c) Multiplying both sides of the given equation by 2^{12}, we obtain

$$1 + 2^1 + 2^2 + \cdots + 2^{12-(t+1)} + 2^{12-t} = n$$

Since the answer to (b) is 6, then $t = 6$ and so we have

$$1 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = n$$

Therefore, $n = 1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$.

Answer: $3, 6, 127$