The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Wednesday, February 24, 2016
(in North America and South America)

Thursday, February 25, 2016
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

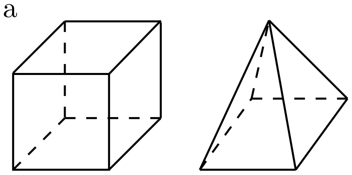
The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

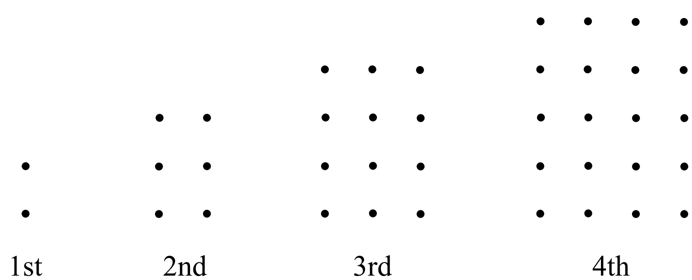
1. If $x = 3$, $y = 2x$ and $z = 3y$, the value of z is
   (A) 8 **(B)** 9 **(C)** 6 **(D)** 18 **(E)** 15

2. A cube has 12 edges, as shown. How many edges does a square-based pyramid have?
   (A) 6 **(B)** 12 **(C)** 8
   (D) 4 **(E)** 10



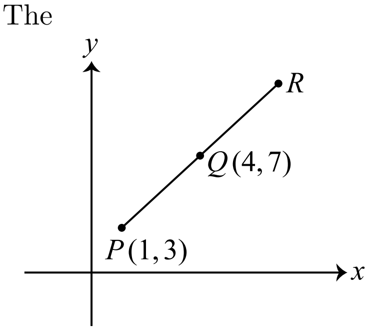
3. The expression $\dfrac{20 + 16 \times 20}{20 \times 16}$ equals
   (A) 20 **(B)** 276 **(C)** 21 **(D)** $\dfrac{9}{4}$ **(E)** $\dfrac{17}{16}$

4. An *oblong number* is the number of dots in a rectangular grid with one more row than column. The first four oblong numbers are 2, 6, 12, and 20, and are represented below:



   What is the 7th oblong number?
   (A) 42 **(B)** 49 **(C)** 56 **(D)** 64 **(E)** 72

5. In the diagram, point Q is the midpoint of PR. The coordinates of R are
   (A) $(2,5)$ **(B)** $(7,11)$ **(C)** $(6,9)$
   (D) $(8,10)$ **(E)** $(9,15)$



6. Carrie sends five text messages to her brother each Saturday and five text messages to her brother each Sunday. Carrie sends two text messages to her brother on each of the other days of the week. Over the course of four full weeks, how many text messages does Carrie send to her brother?
   (A) 15 **(B)** 28 **(C)** 60 **(D)** 80 **(E)** 100

7. The value of $(-2)^3 - (-3)^2$ is
   (A) -17 **(B)** 1 **(C)** -12 **(D)** 0 **(E)** -1

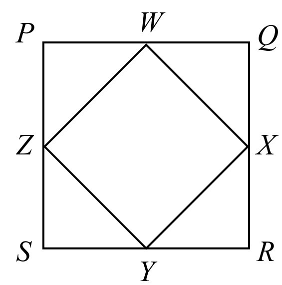
8. If $\sqrt{25-\sqrt{n}}=3$, the value of n is

(A) 4 **(B)** 16 **(C)** 64 **(D)** 484 **(E)** 256

9. If $x\%$ of 60 is 12, then 15% of x is

(A) $\frac{3}{4}$ **(B)** $\frac{1}{3}$ **(C)** 4 **(D)** 3 **(E)** 9

10. In the diagram, square $PQRS$ has side length 2. Points W, X, Y, and Z are the midpoints of the sides of $PQRS$. What is the ratio of the area of square $WXYZ$ to the area of square $PQRS$?

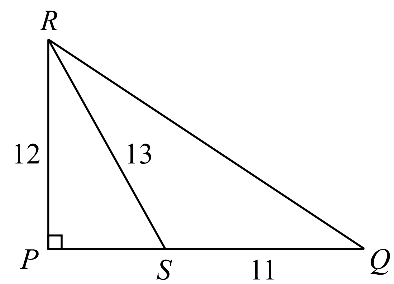


(A) $1:2$ **(B)** $2:1$ **(C)** $1:3$
(D) $1:4$ **(E)** $\sqrt{2}:2$

Part B: Each correct answer is worth 6.

11. In the diagram, $\triangle PQR$ is right-angled at P and $PR=12$. If point S is on PQ so that $SQ=11$ and $SR=13$, the perimeter of $\triangle QRS$ is

(A) 47 **(B)** 44 **(C)** 30
(D) 41 **(E)** 61



12. How many of the positive divisors of 128 are perfect squares larger than 1?

(A) 2 **(B)** 5 **(C)** 1 **(D)** 3 **(E)** 4

13. The numbers $4x, 2x-3, 4x-3$ are three consecutive terms in an arithmetic sequence. What is the value of x?

(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, $3, 5, 7, 9$ are the first four terms of an arithmetic sequence.)

(A) $\frac{3}{4}$ **(B)** $-\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $-\frac{3}{2}$ **(E)** $-\frac{3}{4}$

14. Suppose that a and b are integers with $4<a<b<22$. If the average (mean) of the numbers $4, a, b, 22$ is 13, then the number of possible pairs (a,b) is

(A) 10 **(B)** 8 **(C)** 7 **(D)** 9 **(E)** 6

15. Hicham runs 16 km in 1.5 hours. He runs the first 10 km at an average speed of 12 km/h. What is his average speed for the last 6 km?

(A) 8 km/h **(B)** 9 km/h **(C)** 10 km/h **(D)** 6 km/h **(E)** 12 km/h

16. If $x = 18$ is one of the solutions of the equation $x^2 + 12x + c = 0$, the other solution of this equation is

(A) $x = 216$ **(B)** $x = -6$ **(C)** $x = -30$ **(D)** $x = 30$ **(E)** $x = -540$

17. A total of n points are equally spaced around a circle and are labelled with the integers 1 to n, in order. Two points are called *diametrically opposite* if the line segment joining them is a diameter of the circle. If the points labelled 7 and 35 are diametrically opposite, then n equals

(A) 54 **(B)** 55 **(C)** 56 **(D)** 57 **(E)** 58

18. Suppose that x and y satisfy $\dfrac{x-y}{x+y} = 9$ and $\dfrac{xy}{x+y} = -60$.

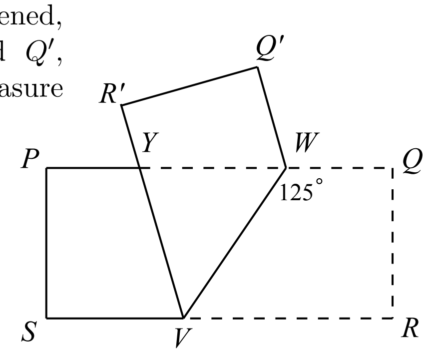
The value of $(x + y) + (x - y) + xy$ is

(A) 210 **(B)** -150 **(C)** 14160 **(D)** -14310 **(E)** -50

19. There are n students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the n students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the n students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the n students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to $n^2 - n$ is

(A) 11 **(B)** 12 **(C)** 20 **(D)** 13 **(E)** 10

20. In the diagram, $PQRS$ represents a rectangular piece of paper. The paper is folded along a line VW so that $\angle VWQ = 125°$. When the folded paper is flattened, points R and Q have moved to points R' and Q', respectively, and $R'V$ crosses PW at Y. The measure of $\angle PYV$ is



(A) 110° **(B)** 100° **(C)** 95°
(D) 105° **(E)** 115°

Part C: Each correct answer is worth 8.

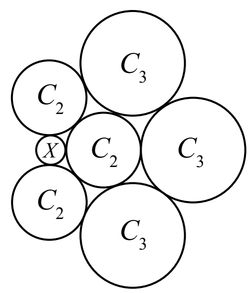
21. Box 1 contains one gold marble and one black marble. Box 2 contains one gold marble and two black marbles. Box 3 contains one gold marble and three black marbles. Whenever a marble is chosen randomly from one of the boxes, each marble in that box is equally likely to be chosen. A marble is randomly chosen from Box 1 and placed in Box 2. Then a marble is randomly chosen from Box 2 and placed in Box 3. Finally, a marble is randomly chosen from Box 3. What is the probability that the marble chosen from Box 3 is gold?

    (A) $\frac{11}{40}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{13}{40}$ **(D)** $\frac{7}{20}$ **(E)** $\frac{3}{8}$

22. If x and y are real numbers, the minimum possible value of the expression $(x+3)^2+2(y-2)^2+4(x-7)^2+(y+4)^2$ is

    (A) 172 **(B)** 65 **(C)** 136 **(D)** 152 **(E)** 104

23. Seven coins of three different sizes are placed flat on a table, arranged as shown in the diagram. Each coin, except the centre one, touches three other coins. The centre coin touches all of the other coins. If the coins labelled C_3 have a radius of 3 cm, and those labelled C_2 have radius 2 cm, then the radius of the coin labelled X is closest to



    (A) 0.615 cm **(B)** 0.620 cm **(C)** 0.610 cm
    (D) 0.605 cm **(E)** 0.625 cm

24. For any real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. For example, $\lfloor 4.2 \rfloor = 4$ and $\lfloor 0.9 \rfloor = 0$.
    If S is the sum of all integers k with $1 \le k \le 999\,999$ and for which k is divisible by $\lfloor \sqrt{k} \rfloor$, then S equals

    (A) 999 500 000 **(B)** 999 000 000 **(C)** 999 999 000
    (D) 998 999 500 **(E)** 998 500 500

25. The set $A = \{1, 2, 3, \ldots, 2044, 2045\}$ contains 2045 elements. A subset S of A is called *triple-free* if no element of S equals three times another element of S. For example, $\{1, 2, 4, 5, 10, 2043\}$ is triple-free, but $\{1, 2, 4, 5, 10, 681, 2043\}$ is not triple-free. The triple-free subsets of A that contain the largest number of elements contain exactly 1535 elements. There are n triple-free subsets of A that contain exactly 1535 elements. The integer n can be written in the form $p^a q^b$, where p and q are distinct prime numbers and a and b are positive integers. If $N = p^2 + q^2 + a^2 + b^2$, then the last three digits of N are

    (A) 202 **(B)** 102 **(C)** 302 **(D)** 402 **(E)** 502

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For students...

Thank you for writing the 2016 Fermat Contest! Each year, more than 220 000 students from more than 60 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Hypatia Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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