



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Galois Contest

(Grade 10)

Thursday, April 16, 2015
(in North America and South America)

Friday, April 17, 2015
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 75 minutes

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Do not open this booklet until instructed to do so.

Number of questions: 4

Each question is worth 10 marks

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks



WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.




- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\dots$ or $1.41\dots$, except where otherwise indicated.

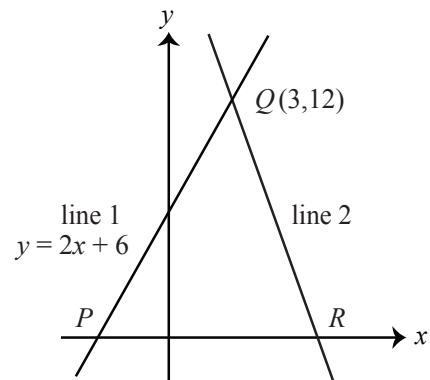
Do not discuss the problems or solutions from this contest online for the next 48 hours.





The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.





NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x -intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

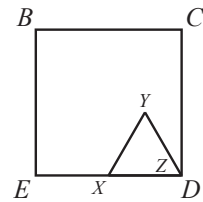
1.  (a) In the diagram, line 1 has equation $y = 2x + 6$ and crosses the x -axis at P . What is the x -intercept of line 1?
 (b) Line 2 has slope -3 and intersects line 1 at $Q(3, 12)$, as shown. Determine the equation of line 2.
 (c) Line 2 crosses the x -axis at R , as shown. Determine the area of $\triangle PQR$.





2. On Wednesday, students at six different schools were asked whether or not they received a ride to school that day.
 (a) At School A, there were 330 students who received a ride and 420 who did not. What percentage of the students at School A received a ride?
 (b) School B has 240 students, of whom 30% received a ride. How many more of the 240 students in School B needed to receive a ride so that 50% of the students in School B got a ride?
 (c) School C has 200 students, of whom 45% received a ride. School D has 300 students. When School C and School D are combined, the resulting group has 57.6% of students who received a ride. If $x\%$ of the students at School D received a ride, determine x .
 (d) School E has 200 students, of whom $n\%$ received a ride. School F has 250 students, of whom $2n\%$ received a ride. When School E and School F are combined, between 55% and 60% of the resulting group received a ride. If n is a positive integer, determine all possible values of n .

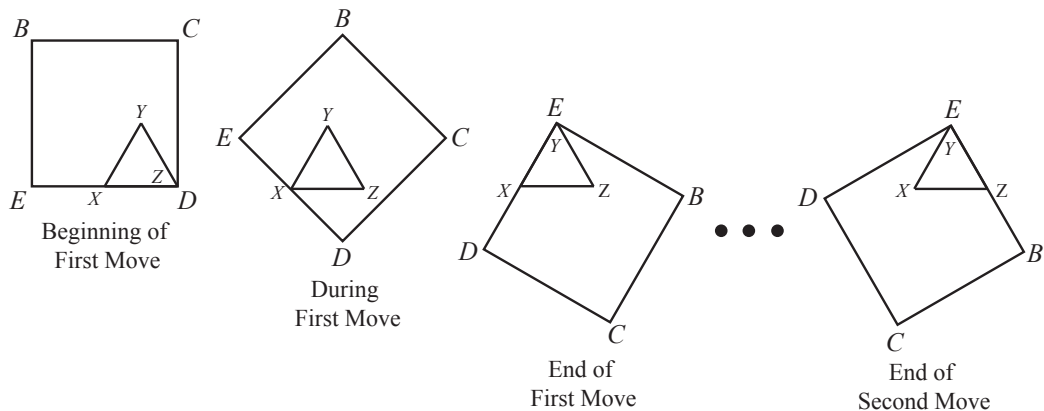
3.  (a) If $n + 5$ is an even integer, state whether the integer n is even or odd.
-  (b) If c and d are integers, explain why $cd(c + d)$ is always an even integer.
-  (c) Determine the number of ordered pairs (e, f) of positive integers where
- $e < f$,
 - $e + f$ is odd, and
 - $ef = 300$.
-  (d) Determine the number of ordered pairs (m, n) of positive integers such that $(m + 1)(2n + m) = 9000$.

4. In the diagram, square $BCDE$ has side length 2. Equilateral $\triangle XYZ$ has side length 1. Vertex Z coincides with D and vertex X is on ED .




-  (a) What is the measure of $\angle YXE$?

-  (b) A move consists of rotating the square clockwise around a vertex of the triangle until a side of the square first meets a side of the triangle. The first move is a rotation about X and the second move is a rotation about Y , as shown in the diagrams. (Note that the vertex of the triangle about which the square rotates remains in contact with the square during the rotation.)



In subsequent moves, the square rotates about vertex Z , then X , then Y , and so on. Determine, with justification, the total number of moves made from the beginning of the first move to when vertex D next coincides with a vertex of the triangle.

-  (c) Determine the length of the path travelled by point E from the beginning of the first move to when square $BCDE$ first returns to its original position (that is, when D next coincides with Z and XZ lies along ED).



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For students...

Thank you for writing the 2015 Galois Contest! Each year, more than 200 000 students from more than 60 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2015.

Visit our website cemc.uwaterloo.ca to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Obtain information about our 2015/2016 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
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- Find your school's contest results