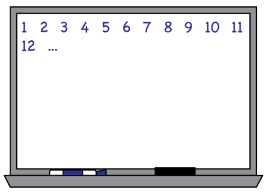

Problem of the Week
Problem C and Solution
Wipe Away 1

Problem

Chetan writes the positive integers from 1 to 200 on a whiteboard. Wassim then erases all the numbers that are multiples of 9. Karla then erases all the remaining numbers that contain the digit 9. How many numbers are left on the whiteboard?

Solution

We first calculate the number of integers that Wassim erases, which is the number of multiples of 9 between 1 and 200. Since $200 = (22 \times 9) + 2$, there are 22 multiples of 9 between 1 and 200. Thus, Wassim erases 22 numbers from the whiteboard.

Now let's figure out how many of the integers from 1 to 200 contain the digit 9. From 1 to 100, these numbers are $9,\ 19, \ldots, 79,\ 89$ as well as $90,\ 91, \ldots, 98,\ 99$. There are 19 of these numbers from 1 to 100. There are another 19 between 101 and 200, which are obtained by adding 100 to each of the numbers from 1 to 100. Therefore, $19 + 19 = 38$ integers from 1 to 200 contain the digit 9.

However, some of the integers that contain the digit 9 are also multiples of 9, so were erased by Wassim. There are 5 of these numbers between 1 and 200: $9,\ 90,\ 99,\ 189$, and 198. Thus, Karla erases $38 - 5 = 33$ numbers from the whiteboard.

Hence, the number of numbers left on the whiteboard is $200 - 22 - 33 = 145$.