Pi Day is an annual celebration of the mathematical constant $\pi$. Pi Day is observed on March 14, since $3, 1, \text{ and } 4$ are the first three significant digits of $\pi$.

Archimedes determined lower bounds for $\pi$ by finding the perimeters of regular polygons inscribed in a circle with diameter of length 1. (An inscribed polygon of a circle has all of its vertices on the circle.) He also determined upper bounds for $\pi$ by finding the perimeters of regular polygons circumscribed in a circle with diameter of length 1. (A circumscribed polygon of a circle has all sides tangent to the circle. That is, each side of the polygon touches the circle in one spot.)

In this problem, we will determine a lower bound for $\pi$ and an upper bound for $\pi$ by considering an inscribed regular hexagon and a circumscribed regular hexagon in a circle of diameter 1.

Consider a circle with centre $C$ and diameter 1. Since the circle has diameter 1, it has circumference equal to $\pi$. Now consider the inscribed regular hexagon $DEBGFA$ and the circumscribed regular hexagon $HIJKLM$.

The perimeter of hexagon $DEBGFA$ will be less than the circumference of the circle, $\pi$, and will thus give us a lower bound for the value of $\pi$. The perimeter of hexagon $HIJKLM$ will be greater than the circumference of the circle, $\pi$, and will thus give us an upper bound for the value of $\pi$.

Using these hexagons, determine a lower and an upper bound for $\pi$.

**Note:** For this problem, you may want to use the following known results:

1. A line drawn from the centre of a circle perpendicular to a tangent line meets the tangent line at the point of tangency.

2. For a circle with centre $C$, the centres of both the inscribed and circumscribed regular hexagons will be at $C$. 

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**Problem of the Week**

**Problem E**

**Pi Hexagons**

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