

## Problem of the Week

### Problem E and Solution

#### Shape Building

#### Problem

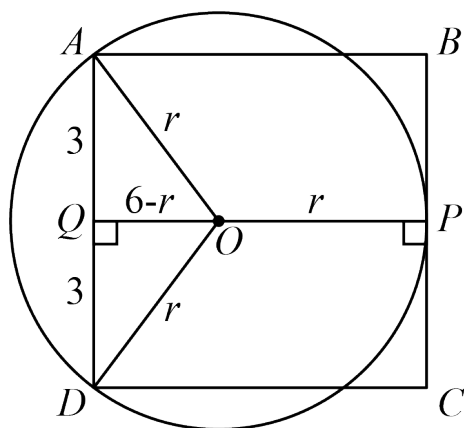
Sina drew square  $ABCD$  with side length 6 cm on a piece of paper and passed the paper to Theo. Theo drew a circle on top of the square so that the circle passes through  $A$  and  $D$ , and the circle is tangent to side  $BC$  at point  $P$ . Determine the radius of the circle.

#### Solution

Let  $O$  be the centre of the circle and  $r$  be the radius. Construct line segment  $PQ$  perpendicular to  $CB$  with  $Q$  on side  $AD$  of the square. Since  $CB$  is tangent to the circle with point of tangency  $P$ ,  $PQ$  must pass through the centre of the circle,  $O$ . Therefore,  $PO = r$ .

Since  $PQ \perp BC$ ,  $PQ \parallel AB$ , and  $PQ = AB = 6$ , then  $QO = PQ - PO = 6 - r$ . Since  $A$  and  $D$  are on the circle,  $AO = DO = r$ .

Using the Pythagorean Theorem,  $AQ^2 = AO^2 - QO^2 = r^2 - (6 - r)^2$  and  $DQ^2 = DO^2 - QO^2 = r^2 - (6 - r)^2$ . Therefore,  $AQ^2 = DQ^2$  and  $AQ = DQ$  follows. Since  $AQ = DQ$  and  $AQ + QD = AD = 6$ , we can substitute to obtain  $AQ + AQ = 2AQ = 6$  or  $AQ = 3$ .



Using the Pythagorean Theorem in  $\triangle AQO$ ,

$$\begin{aligned}AO^2 &= AQ^2 + QO^2 \\r^2 &= 3^2 + (6 - r)^2 \\r^2 &= 9 + 36 - 12r + r^2 \\12r &= 45 \\r &= \frac{45}{12} \\r &= 3.75\end{aligned}$$

Therefore, the radius of the circle is 3.75 cm.