



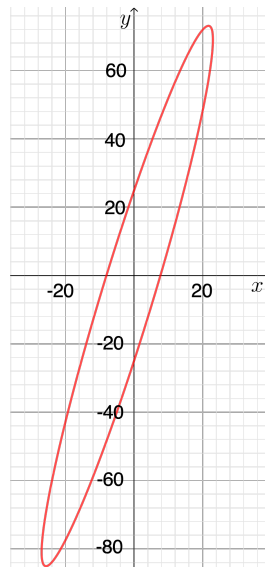
Problem of the Week

Problem E

Points on an Ellipse

The graph of $(x + 1)^2 + (y - 2)^2 = 100$ is a circle with centre $(-1, 2)$ and radius 10.

The graph of $10x^2 - 6xy + 4x + y^2 = 621$ is shown below. The shape of this curve is known as an ellipse.



List all the ordered pairs (x, y) of non-negative integers x and y that satisfy the equation $10x^2 - 6xy + 4x + y^2 = 621$.

NOTE: When solving this problem, it might be useful to use the following idea.

By completing the square,

$$x^2 + y^2 + 2x - 4y = 95$$

can be rewritten as

$$(x + 1)^2 + (y - 2)^2 = 100$$

One solution to this equation is $(x, y) = (5, 10)$.
