



## Problem of the Week

### Problem D and Solution

### Another Average Quiz

#### Problem

On a recent quiz about averages, the following information is known:

- There were three questions on the quiz.
- Each question was worth 1 mark.
- Each question was marked either right or wrong (no part marks).
- 50% of the students got all 3 questions correct.
- 5% of the students got no question correct.
- The class average mark was 2.3 out of 3.

Determine the percentage of students who got exactly 1 question correct and the percentage of students who got exactly 2 questions correct.

#### Solution

##### Solution 1

In this solution, we will use the information given for a class of 100 students, and we will use only one variable.

To determine the average, we must determine the sum of all the marks and divide by the number of students.

Let  $x$  represent the percent of students who got exactly 2 questions correct.

Then  $100 - 50 - 5 - x = (45 - x)$  percent of the students got exactly 1 question correct.

Since 50% of the students got all 3 questions correct, then 50 students each scored 3 marks and earned a total of  $50 \times 3 = 150$  marks.

Since  $x\%$  of the students got exactly 2 questions correct, then  $x$  students each scored 2 marks and earned a total of  $x \times 2 = 2x$  marks.

Since  $(45 - x)\%$  of the students got exactly 1 question correct, then  $(45 - x)$  students each scored 1 mark and earned a total of  $(45 - x) \times 1 = (45 - x)$  marks.

Since 5% of the students got no questions correct, 5 students scored 0 marks and earned a total of  $5 \times 0 = 0$  marks.

The total number of marks earned by the 100 students was  $150 + 2x + (45 - x) + 0 = x + 195$ .



We know that the average mark was 2.3, so

$$\begin{aligned}\frac{x + 195}{100} &= 2.3 \\ x + 195 &= 230 \\ x &= 35 \\ 45 - x &= 10\end{aligned}$$

Therefore, 35% of the students got exactly 2 questions correct and 10% of the students got exactly 1 question correct.

## Solution 2

In this solution, we will use the information given for a class of 100 students, we will use two variables.

To determine the average, we must determine the sum of all the marks and divide by the number of students.

Let  $x$  represent the percent who got exactly 2 questions correct.

Let  $y$  represent the percent who got exactly 1 question correct.

Then,  $50 + x + y + 5 = 100$ , which simplifies to

$$x + y = 45 \tag{1}$$

Since 50% of the students got all 3 questions correct, then 50 students each scored 3 marks and earned a total of  $50 \times 3 = 150$  marks.

Since  $x\%$  of the students got exactly 2 questions correct, then  $x$  students each scored 2 marks and earned a total of  $x \times 2 = 2x$  marks.

Since  $y\%$  of the students got exactly 1 question correct,  $y$  students each scored 1 mark and earned a total of  $y \times 1 = y$  marks.

Since 5% of the students got no questions correct, 5 students scored 0 marks and earned a total of  $5 \times 0 = 0$  marks.

The total number of marks earned by the 100 students was  $150 + 2x + y + 0 = 2x + y + 150$ .

We know that the average score was 2.3, so

$$\begin{aligned}\frac{2x + y + 150}{100} &= 2.3 \\ 2x + y + 150 &= 230 \\ 2x + y &= 80\end{aligned} \tag{2}$$

Subtracting equation (1) from equation (2), we obtain  $x = 35$ . Substituting  $x = 35$  into equation (1), we obtain  $y = 10$ .

Therefore, 35% of the students got exactly 2 questions correct and 10% of the students got exactly 1 question correct.



### Solution 3

In this solution, we will use three variables but we will not assume a class size.

To determine the average, we must determine the sum of all the marks and divide by the number of students.

Let  $n$  represent the number of students who wrote the quiz, where  $n$  is a positive integer.

Let  $x$  represent the percent who got exactly 2 questions correct.

Let  $y$  represent the percent who got exactly 1 question correct.

Then,  $50 + x + y + 5 = 100$  which simplifies to

$$x + y = 45 \tag{1}$$

Since 50% of the students got all 3 questions correct, then  $\frac{50}{100}n$  students each scored 3 marks and earned a total of  $\frac{50}{100}n \times 3 = \frac{150n}{100}$  marks.

Since  $x\%$  of the students got exactly 2 questions correct, then  $\frac{x}{100}n$  students each scored 2 marks and earned a total of  $\frac{x}{100}n \times 2 = \frac{2xn}{100}$  marks.

Since  $y\%$  of the students got exactly 1 question correct, then  $\frac{y}{100}n$  students each scored 1 mark and earned a total of  $\frac{y}{100}n \times 1 = \frac{yn}{100}$  marks.

Since 5% of the students got no questions correct, 5 students scored 0 marks and earned a total of  $5 \times 0 = 0$  marks.

The total number of marks earned by the  $n$  students was  $\frac{150n}{100} + \frac{2xn}{100} + \frac{yn}{100} = \frac{n}{100}(150 + 2x + y)$ .

We know that the average score was 2.3 and  $n$  is a positive integer, so

$$\begin{aligned} \frac{\frac{n}{100}(150 + 2x + y)}{n} &= 2.3 \\ 150 + 2x + y &= 230 \\ 2x + y &= 80 \end{aligned} \tag{2}$$

Subtracting equation (1) from equation (2), we obtain  $x = 35$ . Substituting  $x = 35$  into equation (1), we obtain  $y = 10$ .

Therefore, 35% of the students got exactly 2 questions correct and 10% of the students got exactly 1 question correct.