Problem of the Week

Problem D

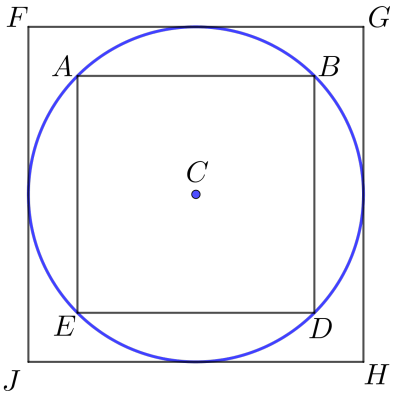
Pi Squares

Pi Day is an annual celebration of the mathematical constant π. Pi Day is observed on March 14, since 3, 1, and 4 are the first three significant digits of π.

Archimedes determined lower bounds for π by finding the perimeters of regular polygons inscribed in a circle with diameter of length 1. (An inscribed polygon of a circle has all of its vertices on the circle.) He also determined upper bounds for π by finding the perimeters of regular polygons circumscribed in a circle with diameter of length 1. (A circumscribed polygon of a circle has all sides tangent to the circle. That is, each side of the polygon touches the circle in one spot.)

In this problem, we will determine a lower bound for π and an upper bound for π by considering an inscribed square and a circumscribed square in a circle of diameter 1.

Consider a circle with centre C and diameter 1. Since the circle has diameter 1, it has circumference equal to π. Now consider the inscribed square $ABDE$ and the circumscribed square $FGHJ$.



The perimeter of square $ABDE$ will be less than the circumference of the circle, π, and will thus give us a lower bound for the value of π. The perimeter of square $FGHJ$ will be greater than the circumference of the circle, π, and will thus give us an upper bound for the value of π.

Using these squares, determine a lower bound and an upper bound for π.

NOTE: For this problem, you may want to use the following known results about circles:

1. For a circle with centre C, the diagonals of an inscribed square meet at 90° at C.
2. For a circle with centre C, the diagonals of a circumscribed square meet at 90° at C.
3. If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.