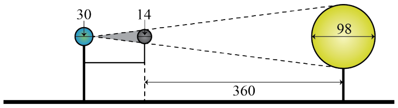
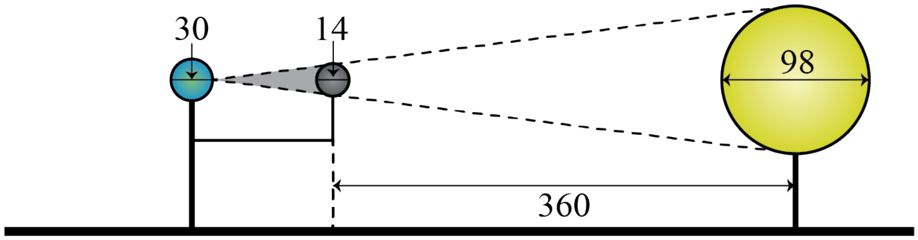


Problem of the Week
Problem D and Solution
Making an Eclipse

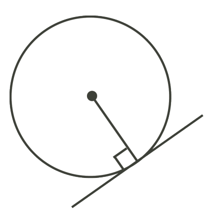
Problem

Quinnen made a model to demonstrate eclipses. She used a spherical LED bulb with diameter 98 mm to represent the Sun, and foam spheres with diameters 30 mm and 14 mm to represent the Earth and the Moon, respectively. The Earth and Sun are attached to a base using metal rods, and the Moon is connected to the Earth's rod by a wire so that it can rotate around the Earth. The centres of the Earth, Moon, and Sun are all the same distance above the base. Quinnen rotates the Moon around the Earth and stops when the Moon is at its closest point to the Sun. In this configuration, her model demonstrates a total solar eclipse. In other words, if you were able to look towards the Sun from the point on the surface of the Earth closest to the Sun, the Moon would completely block the Sun. In this configuration, the distance between centre of the Moon and the centre of the Sun is 360 mm.



Determine the maximum possible distance between the centre of the Earth and the centre of the Moon in Quinnen's model.

Note: You may find the following known result about circles useful:
If a line is tangent to a circle, then the perpendicular to that line at the point of tangency passes through the centre of the circle.



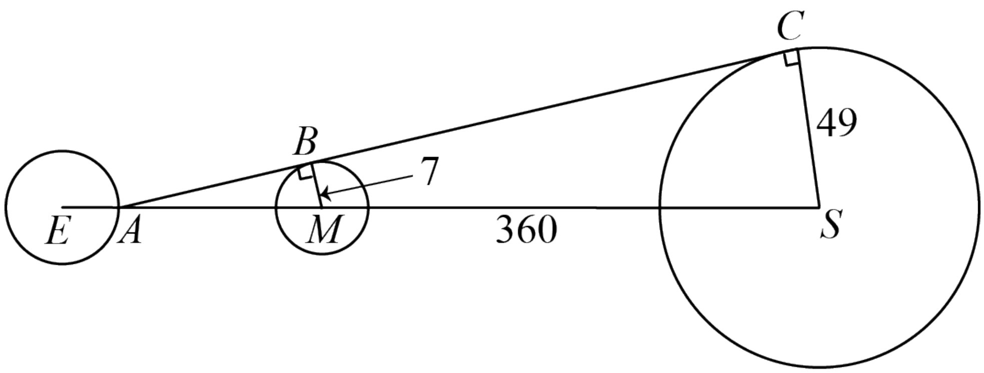
Solution

If the distance between the centre of the Earth and the centre of the Moon is at its maximum, then the area on the surface of the Earth that experiences the total solar eclipse must be as small as possible. Thus, we will assume a singular point on the Earth's surface experiences a total solar eclipse. We will call this point A.

We represent the Earth, Moon, and Sun as circles with centres E, M, and S, respectively. Since the centres of the Earth, Moon, and Sun are all the same distance above the base, E, M, and S all lie on a line. Then A is the point on the circle with centre E that is closest to M, and lies on the line through E, M, and S.

We draw a line from A tangent to both the circle with centre M and the circle with centre S. This line meets the circle with centre M at point B, and the circle with centre S at point C. It follows that BM is a radius of the circle with centre M, so $BM = 14 \div 2 = 7$ mm. Similarly, CS is a radius of the circle with centre S, so $CS = 98 \div 2 = 49$ mm.

Using the given circle result, $\angle ABM = 90^\circ$ and $\angle ACS = 90^\circ$. Since $\angle BAM = \angle CAS$, (same angle), it follows that $\triangle ABM \sim \triangle ACS$.



Using properties of similar triangles,

$$\begin{aligned}\frac{AM}{BM} &= \frac{AS}{CS}\\ \frac{AM}{7} &= \frac{AM+360}{49}\\ 49(AM) &= 7(AM) + 2520\\ 42(AM) &= 2520\\ AM &= 60\end{aligned}$$

Since A is on the circumference of the circle with centre E, $EA = 30 \div 2 = 15$ mm. It follows that $EM = EA + AM = 15 + 60 = 75$ mm. Therefore, the maximum possible distance between the centre of the Earth and the centre of the Moon in Quinnen's model is 75 mm.