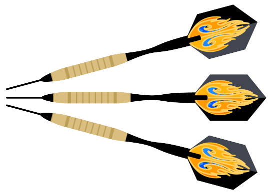

Problem of the Week
Problem D and Solution
Throw to Win

Problem

Kurtis is creating a game for a math fair. They attach n circles, each with radius 1 metre, onto a square wall with side length n metres, where n is a positive integer, so that none of the circles overlap. Participants will throw a dart at the wall and if the dart lands on a circle, they win a prize. Kurtis wants the probability of winning the game to be at least $\frac{1}{2}$.

If they assume that each dart hits the wall at a single random point, then what is the largest possible value of n?

Solution

The area of the square wall with side length n metres is n^2 square metres.

The area of each circle is $\pi(1)^2 = \pi$ square metres. Since there are n circles, the total area covered by circles is $n\pi$ square metres.

If each dart hits the wall at a single random point, then the probability that a dart lands on a circle is equal to the area of the wall covered by circles divided by the total area of the wall. That is,

$$\frac{n\pi \text{ square metres}}{n^2 \text{ square metres}} = \frac{\pi}{n}$$

If this probability must be at least $\frac{1}{2}$, then

$$\begin{aligned}\frac{\pi}{n} &\geq \frac{1}{2} \\ \pi &\geq \frac{n}{2}, \quad \text{since } n > 0 \\ 2\pi &\geq n \\ n &\leq 2\pi \approx 6.28\end{aligned}$$

Thus, since n is an integer, the largest possible value of n is 6.