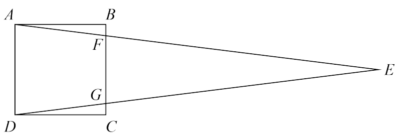


Problem of the Week
Problem C and Solution
Overlapping Shapes 1

Problem

Omar draws square $ABCD$ with side length 4 cm. Jaime then draws $\triangle AED$ on top of square $ABCD$ so that

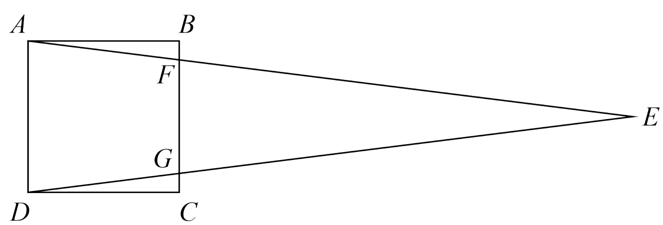
- sides AE and DE meet BC at F and G, respectively,
- FG is 3 cm, and
- the area of $\triangle AED$ is twice the area of square $ABCD$.

Determine the area of $\triangle FEG$.

Solution

Solution 1

In the first solution we will find the area of square $ABCD$, the area of $\triangle AED$, the area of trapezoid $AFGD$, and then use these to calculate the area of $\triangle FEG$.



The area of square $ABCD$ is $4 \times 4 = 16$ cm^2. Since the area of $\triangle AED$ is twice the area of square $ABCD$, it follows that the area of $\triangle AED$ is $2 \times 16 = 32$ cm^2.

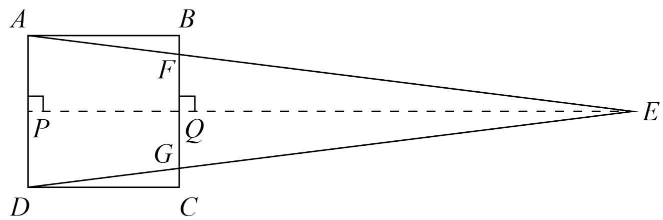
Recall that to find the area of a trapezoid, we multiply the sum of the lengths of the two parallel sides by the height, and divide the product by 2. In trapezoid $AFGD$, the two parallel sides are AD and FG, and the height is the width of square $ABCD$, namely AB.

$$\begin{aligned}\text{Area of trapezoid } AFGD &= (AD + FG) \times AB \div 2 \\ &= (4 + 3) \times 4 \div 2 \\ &= 7 \times 4 \div 2 \\ &= 14 \text{ cm}^2\end{aligned}$$

The area of $\triangle FEG$ is equal to the area of $\triangle AED$ minus the area of trapezoid $AFGD$. Thus, the area of $\triangle FEG$ is $32 - 14 = 18$ cm^2.

Solution 2

We construct an altitude of $\triangle AED$ from E, intersecting AD at P and BC at Q. Since $ABCD$ is a square, we know that AD is parallel to BC. Therefore, since PE is perpendicular to AD, QE is perpendicular to FG and thus an altitude of $\triangle FEG$. In this solution we will find the height of $\triangle FEG$, that is, the length of QE, and then use this to calculate the area of $\triangle FEG$.



The area of square $ABCD$ is $4 \times 4 = 16$ cm^2. Since the area of $\triangle AED$ is twice the area of square $ABCD$, it follows that the area of $\triangle AED$ is $2 \times 16 = 32$ cm^2.

We also know that

$$\begin{aligned} \text{Area } \triangle AED &= AD \times PE \div 2 \\ 32 &= 4 \times PE \div 2 \\ 32 &= 2 \times PE \\ PE &= 32 \div 2 \\ &= 16 \text{ cm} \end{aligned}$$

Since $\angle APQ = 90^\circ$, we know that $ABQP$ is a rectangle, and so $PQ = AB = 4$ cm. We also know that $PE = PQ + QE$. Since $PE = 16$ cm and $PQ = 4$ cm, it follows that $QE = PE - PQ = 16 - 4 = 12$ cm. We can then calculate the area of $\triangle FEG$.

$$\begin{aligned} \text{Area } \triangle FEG &= FG \times QE \div 2 \\ &= 3 \times 12 \div 2 \\ &= 18 \text{ cm}^2 \end{aligned}$$

Therefore, the area of $\triangle FEG$ is 18 cm^2.