



Problem of the Week

Problem B and Solution

It's a Race

Problem

Manish, Diana, Isebel, Ris, and Ji-Yeong are the five runners in a 400 m race. Their friend cheered them on and took a photo partway through the race. The photo shows the following:

- Isebel is in the lead.
- Ji-Yeong has run farther than Ris, but not as far as Manish.
- Diana has two people ahead of her and two people behind her.

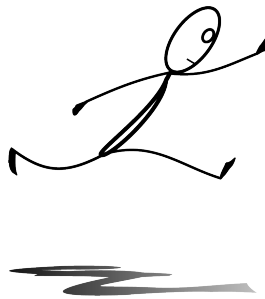
(a) Using the information given, determine the order of the runners in the photo. Fill in the blanks in the list shown below.

START _____, _____, _____, _____, _____ FINISH

(b) The following five fractions represent the fraction of the course that each runner had completed in the photo.

$$\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{3}, \frac{1}{4}$$

Which runner completed each fraction of the course? Show your work using diagrams or equivalent fractions.





Solution

- (a) We will number the positions from 1 to 5, starting on the left. Since Isebel is in the lead, she must be in position 5. Since Diana has two people ahead of her and two people behind her, she must be in position 3. That leaves us with positions 1, 2, and 4. Since Ji-Yeong has run farther than Ris but not as far as Manish, that tells us that Ris must be in position 1, Ji-Yeong must be in position 2, and Manish must be in position 4, as shown.

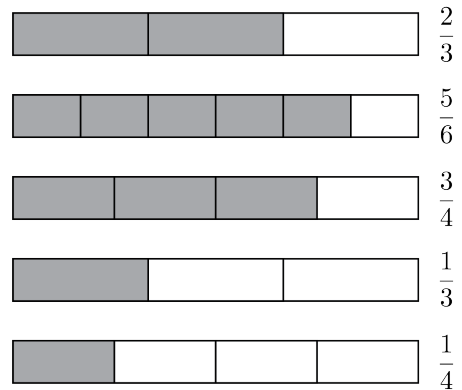
START

Ris, Ji-Yeong, Diana, Manish, Isebel
FINISH

- (b) In order to determine which runner completed each fraction of the course, we must first write the fractions in order from smallest to largest. Then we can match the fractions with the runners in the order from part (a), since the runner who completed the smallest fraction of the course will be closest to the start, and the runner who completed the largest fraction of the course will be closest to the finish.

One way to compare the fractions is using diagrams, as shown.

Since each diagram is the same width, we can compare the shaded part of each diagram to place the fractions in order from smallest to largest. This gives us $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.



Alternatively, we can use equivalent fractions. Using a common denominator of 12, our fractions can be written as follows.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{5}{6} = \frac{10}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{1}{4} = \frac{3}{12}$$

Now we can use the equivalent fractions to place the fractions in order from smallest to largest.

$$\frac{1}{4} = \frac{3}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{5}{6} = \frac{10}{12}$$

Once we have the fractions written in order from smallest to largest, we can match each runner to the fraction of the course they completed as shown.

Runner	Ris	Ji-Yeong	Diana	Manish	Isebel
Fraction of Course Completed	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$