Problem of the Week
Problem A and Solution
Charity Chocolate Cake

Problem

Sam's class is having a bake sale. Sam wants to make cakes with his mom for the fundraiser. Their oven can only fit one cake at a time.

There are four steps to make one cake.

> Step 1: Mix the batter. This takes 10 minutes.
> Step 2: Bake the cake. This takes 35 minutes.
> Step 3: Let the cake cool for 15 minutes.
> Step 4: Frost the cake. This takes 10 minutes.

(a) How long does it take to make one cake from start to finish?

(b) Sam wants to bake two cakes for the bake sale. He starts making the second cake immediately after frosting the first cake. If they need to leave for the bake sale by 4:00 p.m., what is the latest possible time that Sam can start making the first cake?

EXTENSION: You may have noticed that you can make two cakes faster by mixing the batter for the second cake while the first cake is baking, and putting the second cake in the oven immediately after taking the first cake out. Then you can cool and frost the first cake while the second cake is baking. How long would it take from the time you start mixing the batter for the first cake until you finish frosting the second cake with this plan?

Solution

(a) The total time to bake one cake is $10 + 35 + 15 + 10 = 70$ minutes, or 1 hour and 10 minutes.

(b) Since Sam completely finishes baking and frosting one cake before starting the second cake, it will take twice as long to make two cakes. This means it will take $70 \times 2 = 140$ minutes, or 2 hours and 20 minutes. Now we have to count time backwards from 4:00 p.m. to determine when Sam needs to start making the first cake. Two hours before 4:00 p.m. is 2:00 p.m., and 20 minutes before 2:00 p.m. is 1:40 p.m. Therefore, Sam needs to start making the first cake by 1:40 p.m. at the latest.

Another way to calculate what time it is 140 minutes before 4:00 p.m. is to use a clock and move the hands back in time. There are some virtual options for this including https://toytheater.com/clock/.

EXTENSION: To calculate the time it would take to make two cakes with this plan, you could make a timeline as shown below:
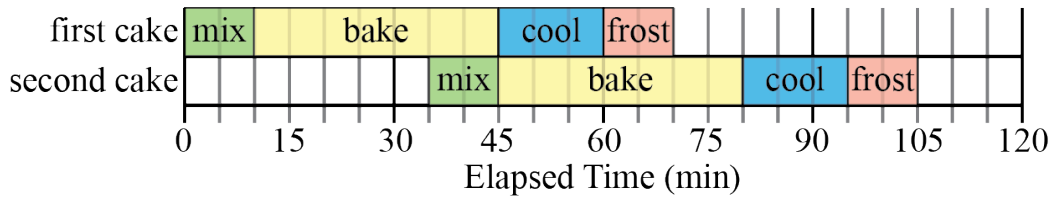
Counting the time from the start until the second cake is finished, we can see it takes 105 minutes, or 1 hour and 45 minutes. Note that the second cake batter can be mixed any time during the 35 minutes the first cake is baking.

Alternatively, you can record the time you are saving when doing something with one cake while the other cake is baking. For the first cake, you're saving the time for cooling and frosting, which is $15 + 10 = 25$ minutes. For the second cake, you're saving the time for mixing the batter, which is 10 minutes. Therefore, in total, you're saving $25 + 10 = 35$ minutes. Since our answer from part (b) was 140 minutes, then with our faster plan we can bake the two cakes in $140 - 35 = 105$ minutes, or 1 hour and 45 minutes.

Teacher's Notes

The extension problem is an example of how *pipelining* can improve production. The basic idea behind pipelining is to break up a task into smaller parts, and complete the smaller parts of separate tasks at the same time (i.e. in parallel).

In this example, mixing the second cake's batter takes place at the same time as the first cake is baking. As well, the first cake is cooling and being frosted at the same time as the second cake is baking. Without pipelining, it takes 2 hours and 20 minutes to finish both cakes. With pipelining, it takes 1 hour and 45 minutes to finish both cakes.

Pipelining is used in lots of places such as assembly lines. It is also used in the design of computer chips to increase the processing speed of your digital devices.