Problem of the Week
Problem D and Solution
Counting Ties

Problem

There are four intramural softball teams at a school, each named after local wildlife: Squirrels, Chipmunks, Raccoons, and Opossums. At the end of the season, each team had played every other team exactly four times. A team earns 3 points for a win, 1 point for a tie, and no points for a loss. The total points earned for each team are as follows.

Team Name	Total Number of Points
Squirrels	12
Chipmunks	14
Raccoons	19
Opossums	22

How many of the games played in the season ended in a tie?

Solution

Since each team played every other team four times, each team played $3 \times 4 = 12$ games. Since there are four teams, a total of $\frac{4 \times 12}{2} = 24$ games were played. Note that we divided by 2 so that we don't double count games. For example, the Squirrels playing the Chipmunks is the same as the Chipmunks playing the Squirrels.

In games where one team won and one team lost, one team earned 3 points and the other team earned 0 points, so a total of 3 points were awarded. In games that ended in a tie, both teams earned 1 point, so a total of 2 points were awarded.

If there were no ties, then 24 games would result in $24 \times 3 = 72$ points being awarded in total. However, $12 + 14 + 19 + 22 = 67$ points were actually awarded in total. Since a total of 3 points were awarded when there was a win and a total of 2 points were awarded when there was a tie, each tie game adds one fewer point to the total number of points than a game where there was a win. It follows that every point below 72 must represent a tie game. Since $72 - 67 = 5$, there must have been 5 tie games.

Since 24 games were played, $24 - 5 = 19$ games resulted in a win. We should check that there is a combination of wins, ties and losses that satisfies the conditions in the problem. One possibility is shown below.

Team Name	Number of Wins	Number of Ties	Number of Losses	Total Points
Squirrels	3	3	6	12
Chipmunks	3	5	4	14
Raccoons	6	1	5	19
Opossums	7	1	4	22
TOTALS	19	10	19	67

In the table, there are a total of 10 ties. This means that 5 games ended in a tie and a total of 10 points were awarded for ties.

Extension: There are 5 other combinations of wins, ties and losses that satisfy the conditions of the problem. Can you find them all?