Problem of the Week
Problem D and Solution
Two Birds

Problem

Katya owns two cockatoos, an older white cockatoo and a younger Galah cockatoo. At present, the sum of the cockatoos' ages is 44 years. In n years, where $n > 0$, the white cockatoo's age will be four times the Galah cockatoo's age. If n is an integer, determine the possible present ages of each cockatoo.

Solution

Let g represent the present age of the Galah cockatoo and w represent the present age of the white cockatoo. Since the sum of their present ages is 44, we have $g + w = 44$ or $w = 44 - g$.

In n years, the Galah cockatoo will be $(g + n)$ years old and the white cockatoo will be $(44 - g + n)$ years old. At that time the white cockatoo will be four times older than the Galah cockatoo. Therefore,

$$4(g + n) = 44 - g + n$$
$$4g + 4n = 44 - g + n$$
$$5g + 3n = 44$$
$$g = \frac{44 - 3n}{5}$$

We are looking for integer values of n so that $44 - 3n$ is divisible by 5.

When $n = 3$, $g = \frac{44-3n}{5} = \frac{44-3(3)}{5} = \frac{35}{5} = 7$. When $g = 7$, $w = 44 - g = 44 - 7 = 37$.

When $n = 8$, $g = \frac{44-3n}{5} = \frac{44-3(8)}{5} = \frac{20}{5} = 4$. When $g = 4$, $w = 44 - g = 44 - 4 = 40$.

When $n = 13$, $g = \frac{44-3n}{5} = \frac{44-3(13)}{5} = \frac{5}{5} = 1$. When $g = 1$, $w = 44 - g = 44 - 1 = 43$.

When $n = 18$, $g = \frac{44-3n}{5} = \frac{44-3(18)}{5} = \frac{-10}{5} = -2$. Since $g < 0$, $n = 16$ does not produce a valid age for the Galah cockatoo. No higher value of n would produce a value of $g > 0$.

No integer values of n between 0 and 18, other than 3, 8, and 13, produce a multiple of 5 when substituted into $44 - 3n$.

If today the white cockatoo is 37 and the Galah cockatoo is 7, then in 3 years the white cockatoo will be 40 and the Galah cockatoo will be 10. The white cockatoo will be four times older than the Galah cockatoo since $4 \times 10 = 40$.

If today the white cockatoo is 40 and the Galah cockatoo is 4, then in 8 years the white cockatoo will be 48 and the Galah cockatoo will be 12. The white cockatoo will be four times older than the Galah cockatoo since $4 \times 12 = 48$.

If today the white cockatoo is 43 and the Galah cockatoo is 1, then in 13 years the white cockatoo will be 56 and the Galah cockatoo will be 14. The white cockatoo will be four times older than the Galah cockatoo since $4 \times 14 = 56$.

Therefore, the possible present ages for the white cockatoo and Galah cockatoo are 37 and 7, or 40 and 4, or 43 and 1.