Problem of the Week Problem B and Solution The Puzzler Returns

Problem

Our superhero The Puzzler is back, seeking your help once more to solve the following number puzzle.

Place each of the numbers 2, 3, 4, 5, 6, 7, 8, 9, and 10 in a different circle in the diagram so that each line of three circled numbers has the same sum.





Can you find more than one possibility for the number that can go in the middle circle? Not printing this page? You can use our interactive worksheet.

Solution

The key discovery for this puzzle is that the middle circle is on every line of three circled numbers. If we were to remove the number in the middle circle from the diagram, then we would be left with four pairs of numbers that each have the same sum. So after choosing the middle number, it must be possible to pair up the remaining eight numbers so that each pair has the same sum.

If the middle number is 2, the other numbers can be paired as follows: 3 + 10, 4 + 9, 5 + 8, and 6 + 7. Each of these sums is 13, so the sum of any line of three circled numbers would be 13 + 2 = 15.

Similarly, if the middle number is 10, the other numbers can be paired as follows: 2+9, 3+8, 4+7, and 5+6. Each of these sums is 11, so the sum of any line of three circled numbers would be 11+10=21.

We can also choose 6 as the middle number. Then the other numbers can be paired as follows: 2 + 10, 3 + 9, 4 + 8, and 5 + 7. Each of these sums is 12, so the sum of any line of three circled numbers would be 12 + 6 = 18.

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It is not possible to choose any other number as the middle number. In each case, if you try pairing up the remaining numbers, you will find that you cannot do so in a way such that each pair has the same sum.

With middle numbers of 2, 6, or 10, there are many possible ways to place the remaining numbers in the diagram. The only condition is that the pairs of numbers with the same sum must be placed on the same line. Some examples are shown.

