Problem of the Week
Problem E and Solution
One More Coin

Problem

A coin collecting club has between 12 and 30 members attend its monthly meeting. For one such meeting, they noticed that all of the members present each had the same number of coins except one member who had one more coin than each of the other members. Between them, the members had a total number of 1000 coins.

How many members attended the meeting?

Solution

Let n represent the number of members present at the meeting. We know that $12 < n < 30$ and n is an integer. Let c represent the number of coins that all but one member had. That member had $c+1$ coins. It follows that $(n-1)$ members had c coins each and one member had $c+1$ coins, producing a total of 1000 coins.

Thus,

$$\begin{aligned}(n-1)\times c + 1\times(c+1) &= 1000\\ nc - c + c + 1 &= 1000\\ nc &= 999\end{aligned}$$

We are looking for two positive integers with a product of 999, with one of the numbers between 12 and 30. The prime factorization of 999 is $3\times3\times3\times37$. We can combine the factors to produce pairs of positive integers whose product is 999. The possibilities are 1 and 999, 3 and 333, 9 and 111, and 27 and 37. The only possible product which gives one factor between 12 and 30 is 27×37.

It follows that there were 27 members present at the last meeting, and 26 of the members had 37 coins each and 1 member had 38 coins. This can be easily verified, as $26\times37+1\times38=1000$.