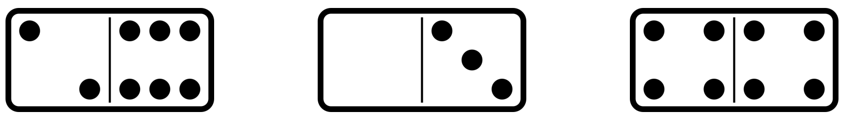
Problem of the Week

Problem D

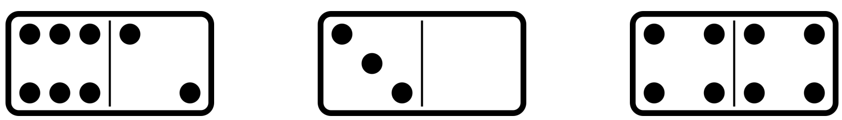
Missing Tile

A domino tile is a rectangular tile with a line dividing its face into two square ends. Each end is marked with a number of dots (also called pips) or is blank.

The first domino shown below is a $[2, 6]$ domino, since there are 2 pips on its left end and 6 pips on its right end. The second domino shown below is a $[0, 3]$ domino, since there are 0 pips on its left end and 3 pips on its right end. The third domino shown below is a $[4, 4]$ domino, since there are 4 pips on its left end and 4 pips on its right end.



We can also rotate the domino tiles. The first domino shown below is a $[6, 2]$ domino, since there are 6 pips on its left end and 2 pips on its right end. However, since this tile can be obtained by rotating the $[2, 6]$ tile, $[6, 2]$ and $[2, 6]$ represent the same domino. Similarly, the second domino shown below is a $[3, 0]$ domino. Again, note that $[3, 0]$ and $[0, 3]$ represent the same domino.



A 2-set of dominoes contains all possible tiles with the number of pips on any end ranging from 0 to 2, with no two dominoes being the same. A 2-set of dominoes has the following six tiles: $[0, 0]$, $[0, 1]$, $[0, 2]$, $[1, 1]$, $[1, 2]$, $[2, 2]$. Notice that the three dominoes $[1, 0]$, $[2, 0]$, and $[2, 1]$ are not listed because they are the same as the three dominoes $[0, 1]$, $[0, 2]$, and $[1, 2]$.

Similarly, a 9-set of dominoes contains all possible tiles with the number of pips on any end ranging from 0 to 9, with no two dominoes being the same.

Drew and Bennett separate a 9-set of dominoes into two piles. Drew counts all of the pips on the dominoes in the first pile. He counts that there are a total of 213 pips. Bennett counts all of the pips on the dominoes in the second pile. He counts that there are a total of 266 pips. They then realize that one domino is missing from the set. Drew also notes that every domino that has the same number of pips on its left and right ends is accounted for. Which domino is missing from the set?