



## Problem of the Week

### Problem D and Solution

#### The Elevator

#### Problem

There are six people in an elevator. The sum of all six of their ages is 190 and the median age is 22. From youngest to oldest, the names of the people in the elevator are Ashish, Brook, Calista, Dipak, Enid, and Freyja.

The elevator stops and Ashish and Enid get off. The mean (average) age of the remaining four people in the elevator is then 30. The elevator then stops again and Brook and Calista get off. The mean age of the remaining two people in the elevator is then 40.

If Ashish is 18 years old, and each person's age is a different positive integer, how old is Freyja?

#### Solution

Let  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$  represent the ages of Ashish, Brook, Calista, Dipak, Enid, and Freyja, respectively.

Since the sum of all six ages is 190, it follows that  $A + B + C + D + E + F = 190$ .

After Ashish and Enid get off the elevator, the mean age of the remaining four people is 30. Thus,

$$\frac{B + C + D + F}{4} = 30$$
$$B + C + D + F = 120$$

Thus, after Ashish and Enid leave, the sum of the ages of the people in the elevator is reduced by  $190 - 120 = 70$ . It follows that  $A + E = 70$ .

After Brook and Calista get off the elevator, the mean age of the remaining two people is 40. Thus,

$$\frac{D + F}{2} = 40$$
$$D + F = 80$$

Thus, after Brook and Calista left, the sum of the ages of the people in the elevator reduced by  $120 - 80 = 40$ . It follows that  $B + C = 40$ .

We are told that Ashish is 18 years old, so  $A = 18$ . Since  $A + E = 70$ , it follows that  $E = 70 - 18 = 52$ .

Since there are six people in total, the median age will be halfway between the two ages in the middle positions when they are arranged in increasing order. Thus, the median will be halfway between  $C$  and  $D$ . Since each age is a different positive integer,  $C$  must be less than the median. Thus,  $C < 22$ . Since  $B + C = 40$  and  $B < C$ , the only possibility is  $C = 21$  and  $B = 19$ .

We know that  $C = 21$ , and the median is halfway between  $C$  and  $D$ . Since the median is 22, we can conclude that  $D = 23$ . Then, since  $D + F = 80$ , it follows that  $F = 80 - 23 = 57$ . Thus, Freyja is 57 years old.