



Problem of the Week

Problem C and Solution

Missing the Fives I

Problem

Bobbi lists the positive integers, in order, excluding all multiples of 5. Her resulting list is

$$1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, \dots$$

How many integers has Bobbi listed just before she leaves out the 2023rd multiple of 5?

Solution

Solution 1

In the list of integers beginning at 1, the 2023th multiple of 5 is $2023 \times 5 = 10\,115$. Thus, Bobbi has listed each of the integers from 1 to 10 114 with the exception of the positive multiples of 5 less than 10 115. Since 10 115 is the 2023rd multiple of 5, Bobbi will not write 2022 multiples of 5.

Therefore, just before Bobbi leaves out the 2023rd multiple of 5, she has listed $10\,114 - 2022 = 8092$ integers.

Solution 2

Beginning at 1, each group of five integers has one integer that is a multiple of 5. For example, the first group of five integers, 1, 2, 3, 4, 5, has one multiple of 5 (namely 5), and the second group of five integers, 6, 7, 8, 9, 10, has one multiple of 5 (namely 10). In Bobbi's list of integers, she leaves out the integers that are multiples of 5, and so in every group of five integers, Bobbi lists four of these integers. Thus, just before Bobbi leaves out the 2023rd multiple of 5, there were 2023 of these groups. Therefore, she has listed $2023 \times 4 = 8092$ integers.