Problem of the Week
Problem C and Solution
The Missing Pieces

Problem
The following information is known about $\triangle PQR$.

- The point $S$ is on side $PR$ and the point $T$ is on side $PQ$.
- The distance from $P$ to $S$ is equal to the distance from $T$ to $Q$.
- The distance from $S$ to $R$ is equal to the distance from $P$ to $T$.
- $\angle PRQ = 40^\circ$ and $\angle PTS = 20^\circ$.

Determine the value of each of the five other interior angles. That is, determine the values of $\angle RPQ$, $\angle STQ$, $\angle TQR$, $\angle RST$, and $\angle PST$.

Solution
First, we let $\angle RPQ$ measure $a^\circ$, $\angle STQ$ measure $b^\circ$, $\angle TQR$ measure $c^\circ$, $\angle RST$ measure $d^\circ$, and $\angle PST$ measure $e^\circ$.

Since $\angle PTQ$ is a straight angle, $20 + b = 180$, and so $b = 160$.

Since $PS = TQ$ and $SR = PT$, it follows that $PS + PR = PT + TQ$, and so $PR = PQ$ and $\triangle PQR$ is isosceles. Therefore $\angle PRQ = \angle PQR$, and so $c = 40$.

Since the angles in a triangle sum to $180^\circ$, in $\triangle PQR$,

\[
\begin{align*}
    a + 40 + c &= 180 \\
    a + 40 + 40 &= 180 \\
    a + 80 &= 180 \\
    a &= 100
\end{align*}
\]
Similarly, in $\triangle PST$,

\[
\begin{align*}
    a + e + 20 &= 180 \\
    100 + e + 20 &= 180 \\
    120 + e &= 180 \\
    e &= 60
\end{align*}
\]

Since $\angle PSR$ is a straight angle,

\[
\begin{align*}
    e + d &= 180 \\
    60 + d &= 180 \\
    d &= 120
\end{align*}
\]

We have determined the value of all the other five interior angles.
\[
\begin{align*}
    \angle RPQ &= a^\circ = 100^\circ, \\
    \angle STQ &= b^\circ = 160^\circ, \\
    \angle TQR &= c^\circ = 40^\circ, \\
    \angle RST &= d^\circ = 120^\circ, \text{ and } \\
    \angle PST &= e^\circ = 60^\circ.
\end{align*}
\]