15   14   n
  12   19

Problem of the Week
Problem C and Solution
Same Same

Problem

The mean (average), the median, and the only mode of the five numbers 15, 12, 14, 19, and n are all equal. Determine the value of n.

Solution

For the five numbers 15, 12, 14, 19, and n to have a single mode, n must equal one of the existing numbers in the list: 15, 12, 14, or 19. It follows that the mean (average), median, and mode must all equal n.

Since there five numbers, and five is an odd number, the median will be equal to the number in the middle position when the five numbers are written in increasing order. When we write the existing numbers in increasing order, we obtain 12, 14, 15, 19. Since n is equal to the median, and must also equal one of the existing numbers, the only possibilities are $n = 14$ or $n = 15$.

If $n = 14$, then the mean of the five numbers is $\dfrac{12 + 14 + 14 + 15 + 19}{5} = 14.8$, which is not equal to 14.

If $n = 15$, then the mean of the five numbers is $\dfrac{12 + 14 + 15 + 15 + 19}{5} = 15$.

Then the mean, median, and mode are all equal to 15. Therefore, the value of n is 15.