Problem of the Week
Problem C and Solution
Thelma's Chips

Problem
Thelma has two piles of bingo chips. In each pile there are green and yellow chips. In one pile, the ratio of the number of green chips to the number of yellow chips is $1 : 2$. In the second pile, the ratio of the number of green chips to the number of yellow chips is $3 : 5$. If Thelma has a total of 20 green chips, then determine the possibilities for the total number of yellow chips.

Solution
Solution 1

In this solution, we first look at all possible combinations of green and yellow chips in the second pile. Since the ratio of the number of green chips to the number of yellow chips in the second pile is $3 : 5$, we know that the number of green chips in this second pile must be a positive multiple of 3. We also know that there are at most 20 green chips in this pile. Thus, the only possible values for the number of green chips in the second pile are 3, 6, 9, 12, 15, and 18. Then, using the fact that the ratio of the number of green chips to the number of yellow chips is $3 : 5$, we can determine the number of yellow chips in the second pile for each case. We can also determine the number of green chips in the first pile by subtracting the number of green chips in the second pile from 20. Finally, we can determine the number of yellow chips in the first pile by multiplying the number of green chips in the first pile by 2. This information for each case is summarized in the table below.

Number of green chips in pile 2	Number of yellow chips in pile 2	Number of green chips in pile 1	Number of yellow chips in pile 1	Total number of yellow chips
3	5	$20 - 3 = 17$	34	$5 + 35 = 39$
6	10	$20 - 6 = 14$	28	$10 + 28 = 38$
9	15	$20 - 9 = 11$	22	$15 + 22 = 37$
12	20	$20 - 12 = 8$	16	$20 + 16 = 36$
15	25	$20 - 15 = 5$	10	$25 + 10 = 35$
18	30	$20 - 18 = 2$	4	$30 + 4 = 34$

Therefore, there are six possible values for the total number of yellow chips. There could be 34, 35, 36, 37, 38, or 39 yellow chips in total.

Solution 2

Let a represent the number of green chips in the first pile, where a is a positive integer. Since the ratio of green chips to yellow chips in this pile is $1:2$, then there are $2a$ yellow chips in this pile.

Let $3b$ represent the number of green chips in the second pile, where b is a positive integer. Since the ratio of green chips to yellow chips in this pile is $3:5$, then there are $5b$ yellow chips in this pile.

In total, there are 20 green chips, so $a + 3b = 20$. Also, the total number of yellow chips is equal to $2a + 5b$.

We consider all the possible values for positive integers a and b that satisfy the equation $a + 3b = 20$. Using these values of a and b, we can then find the possible values of $2a + 5b$, and hence the possible values for the total number of yellow chips.

The results are summarized in the table below.

$a+3b$	b	a	$2a+5b$
20	1	17	39
20	2	14	38
20	3	11	37
20	4	8	36
20	5	5	35
20	6	2	34

Therefore, there are six possible values for the total number of yellow chips. There could be 34, 35, 36, 37, 38, or 39 yellow chips in total.