Problem of the Week Problem B and Solution Painting a Birdhouse

Problem

Bird feeders come in many shapes and sizes. Meera has one with a pentagonal base, five identical rectangular sides, and five identical triangles that meet at a point forming the roof. Each rectangular side has a width of 10 cm, a height of 15 cm, and a square window of side length 8 cm. Each triangle has a height of 12 cm and its base lines up with the top width of one of the rectangular sides.



- (a) What is the total area of the five windows in the feeder?
- (b) Meera has decided to paint the outer faces of the triangular roof segments and the outer sides of the feeder (except the windows), but not the base. What is the total surface area of the parts of the feeder Meera intends to paint?
- (c) Suppose you can purchase a 100 mL can of paint for \$3.50 which will cover 10 000 cm² of surface area. If Meera does two coats of paint on each pentagonal bird feeder, how many complete pentagonal bird feeders can be painted by one of these cans of paint?

Solution

- (a) Since each of the five windows is an 8 cm square of area $8 \times 8 = 64 \text{ cm}^2$, the total area of the windows is $5 \times 64 = 320 \text{ cm}^2$.
- (b) The parts of the feeder to be painted are the five rectangular borders around the windows plus the five triangular roof segments.

The area of one rectangular border is the area of the outer rectangle minus the area of the square window. Since the area of the outer rectangle is $10 \times 15 = 150 \text{ cm}^2$, and the area of the square window is 64 cm^2 , the area of one rectangular border is $150 - 64 = 86 \text{ cm}^2$.

There are five of these borders and so their total area is $5 \times 86 = 430 \text{ cm}^2$.

The area of one triangular roof segment is $\frac{1}{2} \times 10 \times 12 = 60 \text{ cm}^2$. There are five of these triangles and so their total area is $5 \times 60 = 300 \text{ cm}^2$.

Thus, the total area to be painted is $430 + 300 = 730 \text{ cm}^2$.





(c) Two coats of paint on one feeder will require paint for $2 \times 730 = 1460 \text{ cm}^2$. Thus, Meera can paint $10\,000 \div 1460 \approx 6.8$ birdhouses. Therefore, Meera can paint 6 complete birdhouses using one can of paint.