Problem of the Week
Problem B and Solution
Mystery Dimensions

Problem
Eight congruent rectangles are arranged to form a larger rectangle as shown.

(a) If the congruent rectangles each have a length of 6 cm and a width of 3 cm, what is the perimeter of the larger rectangle?

(b) Suppose that the congruent rectangles each have a longer side of length $L$ cm and a shorter side of length 4 cm. Suppose also that the perimeter of the larger rectangle is 64 cm.

(i) What is the value of $L$?

(ii) What is the area of one of the eight congruent rectangles?

EXTENSION: Can you solve part (b) without knowing that the length of the shorter side of each rectangle is 4 cm? If so, how?

Solution

(a) Since each rectangle has a length of 6 cm and a width of 3 cm, the larger rectangle must have sides of lengths $6 + 6 = 12$ cm and $3 + 6 + 3 = 12$ cm. Thus, the perimeter of the larger rectangle is $12 + 12 + 12 + 12 = 48$ cm.

(b)

(i) Since each rectangle has a longer side of length $L$ cm and shorter side of length 4 cm, we can label our diagram to find the dimensions of the larger rectangle. Using this, we determine that the lengths of the sides of the larger rectangle are $L + L = 2L$ and $4 + L + 4 = L + 8$. Since we know the perimeter of the larger rectangle is 64 cm, we can write the following equation.
\[2L + 2L + (L + 8) + (L + 8) = 64\]
\[6L + 16 = 64\]
\[6L = 64 - 16\]
\[6L = 48\]

Since \(6 \times 8 = 48\), it follows that \(L = 8\) cm.

(ii) The area of a rectangle is equal to its length times its width. Thus, the area of each congruent rectangle is \(8 \times 4 = 32\) cm\(^2\).

**Extension Solution:**

If we ignore the two rectangles on the top and the two rectangles on the bottom, we can see that two rectangles placed on top of each other horizontally have a height of \(L\). Therefore, the shorter side of each rectangle equals half its longer side, or \(\frac{L}{2}\). We can label our diagram to find the dimensions of the larger rectangle.

Using this, we determine that the larger rectangle has sides of length \(L + L = 2L\) and \(\frac{L}{2} + L + \frac{L}{2} = 2L\). So the larger rectangle is actually a square with side length \(2L\). Since we know its perimeter is 64 cm, it follows that \(2L + 2L + 2L + 2L = 64\), or \(8L = 64\). Since \(8 \times 8 = 64\), it follows that \(L = 8\) cm. So, we can solve this problem without knowing the width of each rectangle.