



Problem of the Week

Problem E and Solution

Mixture of Three

Problem

Kanza is making and selling trail mix. She makes three different blends, each consisting of a mixture of cashews, dark chocolate, and almonds. All of these blends are sold at the same price of \$18 per kg. If she mixes cashews, dark chocolate, and almonds in the ratio of 1 : 1 : 1, by mass, then she makes a profit of 20%. If she mixes cashews, dark chocolate, and almonds in the ratio of 3 : 2 : 1, by mass, then she makes a profit of 8%. If she mixes cashews, dark chocolate, and almonds in the ratio of 1 : 4 : 2, by mass, then she makes a profit of 26%.

- What price, in dollars per kg, does Kanza pay for each of the cashews, dark chocolate, and almonds?
- What percentage of a profit would she make if she mixes cashews, dark chocolate, and almonds in the ratio of 2 : 3 : 4, by mass?

Solution

Let c be the price Kanza pays for cashews, in dollars per kg.

Let d be the price Kanza pays for dark chocolate, in dollars per kg.

Let a be the price Kanza pays for almonds, in dollars per kg.

Consider the blend where she mixes cashews, dark chocolate, and almonds in the ratio of 1 : 1 : 1, by mass. In 1 kg of this blend, $\frac{1}{3}$ kg is cashews, $\frac{1}{3}$ kg is dark chocolate, and $\frac{1}{3}$ kg is almonds. Also, 1 kg of this blend will cost Kanza $\frac{1}{3}c + \frac{1}{3}d + \frac{1}{3}a$ and will be sold for \$18. Since she makes a profit of 20%, we have

$$1.2 \left(\frac{1}{3}c + \frac{1}{3}d + \frac{1}{3}a \right) = 18$$

Multiplying by 3, we obtain

$$1.2(c + d + a) = 54$$

Dividing by 1.2, we obtain

$$c + d + a = 45 \tag{1}$$

Consider the blend where she mixes cashews, dark chocolate, and almonds in the ratio of 3 : 2 : 1, by mass. In 1 kg of this blend, $\frac{1}{2}$ kg is cashews, $\frac{1}{3}$ kg is dark chocolate, and $\frac{1}{6}$ kg is almonds. Also, 1 kg of this blend will cost Kanza $\frac{1}{2}c + \frac{1}{3}d + \frac{1}{6}a$ and will be sold for \$18. Since she makes a profit of 8%, we have

$$1.08 \left(\frac{1}{2}c + \frac{1}{3}d + \frac{1}{6}a \right) = 18$$

Multiplying by 6, we obtain

$$1.08(3c + 2d + a) = 108$$

Dividing by 1.08, we obtain

$$3c + 2d + a = 100 \tag{2}$$



Consider the blend where she mixes cashews, dark chocolate, and almonds in the ratio of $1 : 4 : 2$, by mass. In 1 kg of this blend, $\frac{1}{7}$ kg is cashews, $\frac{4}{7}$ kg is dark chocolate, and $\frac{2}{7}$ kg is almonds. Also, 1 kg of this blend will cost Kanza $\frac{1}{7}c + \frac{4}{7}d + \frac{2}{7}a$ and will be sold for \$18. Since she makes a profit of 26%, we have

$$1.26 \left(\frac{1}{7}c + \frac{4}{7}d + \frac{2}{7}a \right) = 18$$

Multiplying by 7, we obtain

$$1.26(c + 4d + 2a) = 126$$

Dividing by 1.26, we obtain

$$c + 4d + 2a = 100 \tag{3}$$

We now need to solve the following system of equations.

$$c + d + a = 45 \tag{1}$$

$$3c + 2d + a = 100 \tag{2}$$

$$c + 4d + 2a = 100 \tag{3}$$

First, subtracting equation (1) from equation (2) gives

$$2c + d = 55 \tag{4}$$

Second, doubling equation (1) and then subtracting equation (3) gives

$$c - 2d = -10 \tag{5}$$

We will now use equations (4) and (5) to solve for c and d . Doubling equation (4) and then adding equation (5) gives

$$5c = 100$$

or

$$c = 20$$

Substituting $c = 20$ into equation (4), we get $2(20) + d = 55$ or $d = 15$.

Substituting $c = 20$ and $d = 15$ into equation (1), we find $a = 10$.

- (a) Therefore, Kanza pays \$20 per kg for cashews, \$15 per kg for dark chocolate, and \$10 per kg for almonds.
- (b) Suppose she mixes cashews, dark chocolate, and almonds in the ratio of $2 : 3 : 4$, by mass. Then, in 1 kg of the blend, $\frac{2}{9}$ kg is cashews, $\frac{3}{9}$ kg is dark chocolate, and $\frac{4}{9}$ kg is almonds. Also, 1 kg of this blend will cost Kanza $\frac{2}{9}c + \frac{3}{9}d + \frac{4}{9}a = \frac{2}{9}(20) + \frac{3}{9}(15) + \frac{4}{9}(10) = \frac{125}{9}$ dollars.

She sells 1 kg of this blend for \$18. Since $18 \div \frac{125}{9} = 1.296$, the percentage profit that Kanza makes on this mixture is 29.6%.