Problem of the Week
Problem D and Solution
Not As It Seems

Problem

In the following table, the letters a, b, c, d, and e represent unknown numbers.

	Column 1	Column 2	Column 3
Row 1	75	b	83
Row 2	76	80	d
Row 3	a	81	85
Row 4	78	c	e

At a first glance, the numbers in the table may appear to follow a very predictable pattern. However, we need the columns and rows to follow the following rules:

1. The sum of the numbers in each of the four rows is the same.

2. The sum of the numbers in each of the three columns is the same.

3. The sum of any row does not equal the sum of any column.

Determine the values of a, b, c, d, and e.

Solution

The final answer is $a = 23$, $b = 31$, $c = 60$, $d = 33$, and $e = 51$. We will give our solution below.

Each of the first three rows has two known values and one unknown value. We also know that the sum of each row is the same.

Therefore,

$$\text{Sum of Row 1} = \text{Sum of Row 2}$$
$$75 + b + 83 = 76 + 80 + d$$
$$b + 158 = 156 + d$$
$$d = b + 2$$

Also,

$$\text{Sum of Row 1} = \text{Sum of Row 3}$$
$$75 + b + 83 = a + 81 + 85$$
$$b + 158 = a + 166$$
$$a = b - 8$$

Replacing a with $b - 8$ and d with $b + 2$, we get the following grid:

	Column 1	Column 2	Column 3
Row 1	75	b	83
Row 2	76	80	$b + 2$
Row 3	$b - 8$	81	85
Row 4	78	c	e

Now, each column has the same sum. We will use this fact to find the values of c and e.

$$\text{Sum of Column 1} = \text{Sum of Column 2}$$
$$75 + 76 + (b - 8) + 78 = b + 80 + 81 + c$$
$$b + 221 = b + c + 161$$
$$c = 60$$

Also,

$$\text{Sum of Column 1} = \text{Sum of Column 3}$$
$$75 + 76 + (b - 8) + 78 = 83 + (b + 2) + 85 + e$$
$$b + 221 = b + e + 170$$
$$e = 51$$

Since we know $c = 60$ and $e = 51$, we can determine the row sum using the fourth row. The row sum is $78 + 60 + 51 = 189$. We can use this sum to determine the value of b.

From Row 1, $75 + b + 83 = 189$ and $b = 31$ follows.

We know that $d = b + 2$, so $d = 33$. Also, we know that $a = b - 8$, so $a = 23$.

Therefore, $a = 23$, $b = 31$, $c = 60$, $d = 33$, and $e = 51$. From here, one can easily verify that each row sums to 189 and each column sums to 252.